

OPTIMIZING AND THINNING PLANAR ARRAY USING CHEBYSHEV DISTRIBUTION AND IMPROVED PARTICLE SWARM OPTIMIZATION

Noor Salem Alshdaifat¹ and Mohammed Hussein Bataineh²

Communications Engineering Department, Hijjawi Faculty of Engineering Technology,
Yarmouk University, Irbid, Jordan

noor_88m99@yahoo.com¹, mohbat@yu.edu.jo²

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ABSTRACT

In this paper, a new approach to the optimum solution based on the Chebyshev distribution for planar array and an improved particle swarm optimization (IPSO) will be considered. The current excitation of each element is used as the optimization factor with an aim to suppress the side lobe level (SLL) and reduce the half power beam width (HPBW) with prescribed nulls. Chebyshev distribution is used to define IPSO search space. The same array is then thinned to find the best distribution of the active, or on, elements in order to obtain the desired requirements.

KEYWORDS

Planar array, Chebyshev distribution, Improved particle swarm optimization, Side lobe level suppression, Half power beam width.

1. INTRODUCTION

Wireless communication demands for high gain or directional antennas to decrease interference from other applications [1] necessitate a high gain and precise directional antenna to concentrate the energy in the desired direction. The obvious solution to these demands is to reduce the HPBW of the radiation pattern. This can be achieved by replacing the antenna elements in a certain geometry and improving its performance by proper current and/or phase excitation. Recently, the optimization for solving antenna synthesis has revived with the evolutionary and nature inspired algorithms [2]-[3]. This has vividly improved wireless communication systems by using smart antennas [4]. Smart antennas have the ability to automatically direct the main lobe towards the desired user while directing the nulls in the direction of interference so that side lobes towards other users are minimized. In recent studies, researchers focused on various types of optimization to synthesize different array geometries [5]-[7].

Recent studies for planar array focus on new optimization techniques [8] in order to find their efficiency in finding the optimum solution for an array antenna to achieve the desired goal. Other researchers concentrate on the concept of array thinning. Thinned array means that some of uniformly spaced or periodic array elements are turned on while some of them are turned off where the element positions are fixed. This will lead to reduce the cost while maintaining the same characteristics of the radiation pattern when the filled array of the same size is on.

Planar array with uniform current excitation can be thinned to shape the radiation pattern in order to have a lower SLL and reduced HPBW. It is also possible to place nulls in certain directions. Linear and planar thinned arrays have become of interest to researchers in recent years [9]-[10],

because they can shape the radiation pattern without changing the current excitation. Keizer [11] has used the concept of array thinning and applied it on tapered planar arrays using Fourier transform. The same concept will be used in our treatment of IPSO. The algorithm will first find the optimum current for the desired requirements, then the algorithm will find the best distribution of the active (on) elements to maintain the desired requirements of the filled array with tapered current excitation.

In this paper, a new approach to the optimum solution for planar array based on Chebyshev distribution will be considered. Here, Chebyshev distribution defines IPSO search space as the upper and the lower limit based on the desired SLL. The algorithm seeks optimum excitation for the given objective function to obtain the desired radiation pattern in the new defined search space. This will enable the algorithm to search for the optimum solution and exclude the initial randomness of defining the search space; hence leading to reduce the search time and enhance the obtained results. After finding the optimum current, the algorithm will randomly apply thinning to the optimized array.

2. CHEBYSHEV DISTRIBUTIONS FOR PLANAR ANTENNA ARRAY SYNTHESIS

2.1 Planar Arrays

A planar array gives more degree of freedom to control the radiation pattern. Here, the radiation pattern is more flexible so that a planer array has some advantages over a linear array and encompasses more applications such as remote sensing, tracking radar, search radar and communications, to name just a few.

Planar arrays can be considered as an extension to linear arrays. A planer array consists of N elements in the y -axis and M elements in the x -axis spread over a rectangular grid as shown in Figure 1, where it can be seen as an M linear array of N elements or as an N linear array of M elements.

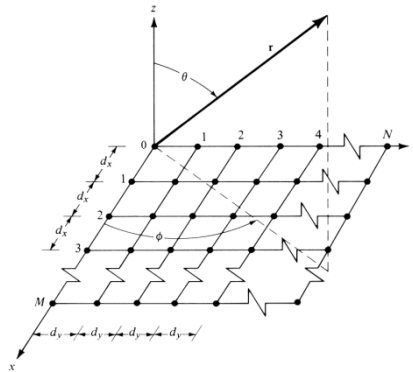


Figure 1. $M \times N$ planar array geometry with equal spacing in the x and y directions [12].

To formulate an equation of the array factor for planar arrays, there are two methods. The first is to use the principle of pattern multiplication; i.e., multiplying the array factor of the M linear array at the x -axis by the N linear array at the y -axis, which takes the form [12]:

$$AF = AF_x \cdot AF_y \quad (1)$$

$$AF_x = \sum_{m=0}^{M-1} w_m e^{jm\psi_x} \quad (1.a)$$

$$AF_y = \sum_{n=0}^{N-1} w_n e^{jn\psi_y} \quad (1.b)$$

Here, $\Psi_x = kd_x \hat{a}_x \cdot \hat{a}_r = kd_x \sin \theta \cos \phi + \beta_x$, $\Psi_y = kd_y \hat{a}_y \cdot \hat{a}_r = kd_y \sin \theta \sin \phi + \beta_y$, w_m and w_n is the current excitation of the m^{th} element and the n^{th} element at the x -axis and the y -axis, respectively, d_x and d_y are the uniform spacing between the elements in the x -axis and y -axis respectively, β_x and β_y are the phase excitation (relative to the array center) of the m^{th} element and the phase excitation (relative to the array center) of the n^{th} element, respectively.

The other method, that is adopted in this paper, is the same as Eq. (1), but it differs only in considering the mn^{th} current to be independent and not restricted to the value of the multiplication [8]. The array factor of the planar array will be given in the form:

$$AF = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} w_{mn} e^{jm\Psi_x} e^{jn\Psi_y} \quad (2)$$

In Eq. (2), the variables are increased, increasing the complexity of the optimization process, but at the same time additional control is given over the shape of the radiation pattern.

In order to steer the main beam to (θ_o, ϕ_o) direction, the phase excitations (β_x and β_y) need to be changed to:

$$\beta_x = -kd_x \sin \theta_o \cos \phi_o \quad (3.a)$$

$$\beta_y = -kd_y \sin \theta_o \sin \phi_o \quad (3.b)$$

To simplify the calculation and reduce the time consumed when taking all the azimuth angels into account (approximately 13.48 hours), one can take a single angle ($\phi = 0^\circ$) to simplify the radiation pattern of Eq. (2). The approximated far field array factor will be in the form:

$$AF = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} w_{mn} e^{j(m-1)(kd_x \sin \theta)} \quad (4)$$

The next step in the design process is to formulate the objective function. The fitness function is formulated such that the maximum peaks of the SLL of the obtained array factor are restricted not to exceed a predefined level (ζ). This fitness function may provide the solution to have the desired SLL, but it may affect the desired HPBW, so the difference between the desired beamwidth (BW_d) and the obtained one (BW) is calculated, and it sets nulls at the interference angles θ_n and make them equal to q ; i.e., a desired null depth.

$$\text{Costfunction} = \sum_m (AF(\theta_{msl}) - \zeta)^2 + (BW - BW_d)^2 + \sum_n (AF(\theta_n) - q)^2; \quad (5)$$

where m is the number of the SLL peaks and θ_{msl} represents the angles at these local maxima of the AF, BW_d is calculated using a formula given by [13].

2.2 Chebyshev Distribution

Chebyshev array was first introduced by Dolph [14], then it was examined by other researchers [15]-[16]. Chebyshev array has proven to produce the smallest HPBW for a given side lobe level or the lowest SLL for a given HPBW. Hence, it is usually referred to as an optimal array. The side lobes in Chebyshev array are equal in level. The idea of Chebyshev array is based on the relation between the cosine functions and the Chebyshev polynomials, where a symmetric amplitude excitation is usually assumed. To design a Chebyshev array, one needs the number of array elements and the desired side lobe level. Barbieri [16] analyzes the procedure to find the excitations for Chebyshev array and present a formula to directly calculate them.

3. PARTICLE SWARM OPTIMIZATION AND IMPROVED PARTICLE SWARM OPTIMIZATION

Particle swarm optimization (PSO) [17] is a robust heuristic multi-dimensional global optimization method, which is based on swarm intelligence. PSO starts by randomly distributing the agents within the search space, then evaluating the fitness of each agent using the fitness function, where each particle knows its best value (p_{best}) and each particle knows the best value

so far in the entire group (g_{best}) among all p_{best} . After that, each particle tries to modify its position based on the distance between the current position and p_{best} and the distance between the current position and g_{best} . The velocity and position of each particle (element) are changed according to:

Speed update:

$$v_{in}^{k+1} = wv_{in}^k + c_1r_1^k(p_{best_{in}}^k - x_{in}^k) + c_2r_2^k(g_{best_{in}}^k - x_{in}^k); \quad (6)$$

Position update:

$$x_{in}^{k+1} = x_{in}^k + v_{in}^{k+1}; \quad (7)$$

where v_{in}^k is the speed and x_{in}^k is the position of the i^{th} particle in the d^{th} dimension at its k^{th} iteration, the parameter w is a number in the range $[0,1]$ called the "inertia weight", which controls the current speed of the particle depending on its previous speed, r_1^k and r_2^k are two uniformly distributed random numbers in the interval $[0,1]$, w indicates the weight by which the particle's current velocity depends on its previous velocity. It was experimentally found in [17] that if the value of w is linearly damped with iterations, it will converge faster. $p_{best_{in}}^k$ and $g_{best_{in}}^k$ are the personal best position of the i^{th} particle and the global best position of the swarm, respectively.

The two coefficients c_1 and c_2 represent the relative weights of the personal best position *versus* the global best position, which regulate the length when flying to the most optimal particle of the whole swarm and to the most optimal individual particle, c_1 and c_2 are set to be 2. Figure 2 is a flow chart that illustrates the process of PSO algorithm.

3.1 Improved Particle Swarm Optimization

Improved particle swarm optimization [18] is a modification to the original PSO with the aim to enhance its searching ability. It has the same strong features of PSO, but has a better searching procedure which consequently makes the algorithm converge faster; the only change here is made on how to calculate velocity.

In Eq. (6), r_1 and r_2 are two uniformly distributed random numbers in the interval $[0,1]$ which are independent of each other. There will be a problem in searching for the optimal solution if both of them were large. In this case, the agent will be taken far from the local optimum. If both of them were small, the personal and social experiences are not fully used and the convergence speed of the technique is reduced.

Instead of having two independent random numbers, one random number, say r_1 , is used [18]. r_2 in Eq. (6) is set as $(1-r_1)$. When r_1 is large, $(1-r_1)$ will be small and *vice versa*. Another random number r_2 is added to have more control of the balance between the local and global search. The new formula to calculate the particle velocity is [18]:

$$v_{in}^{k+1} = r_2^k v_{in}^k + (1 - r_2^k) c_1 r_1^k (p_{best_{in}}^k - x_{in}^k) + (1 - r_2^k) c_2 (1 - r_1^k) (g_{best_{in}}^k - x_{in}^k); \quad (8)$$

where the position is calculated as in Eq. (7). Figure 2 summarizes the steps used in PSO and IPSO algorithms.

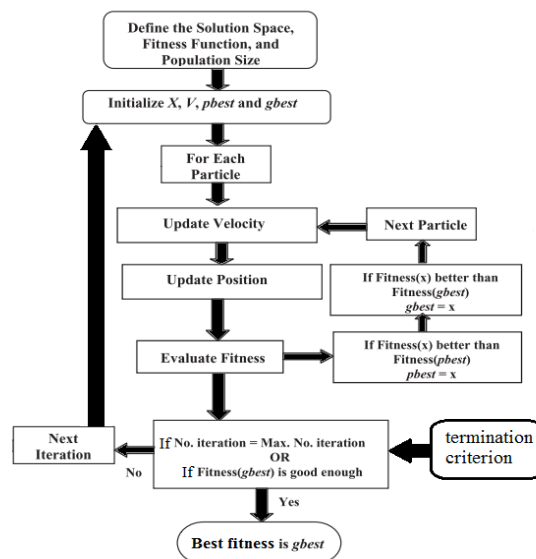


Figure 2. Flow chart of PSO and IPSO algorithm.

4. SIMULATION RESULTS

4.1 Planar Antenna Array Design Using IPSO

In this section, the current for optimizing 10×10 planar array will be demonstrated. The algorithm is implemented using MATLAB. The design parameters are illustrated in Table 1. The search space has changed, the new values are based on the current distribution of Chebyshev planar array [19].

Table 1. IPSO design parameters for a symmetric planar array.

Symbol	Quantity	Value
N	Number of elements	100
c_1 and c_2	Speeding figure	2
P_{size}	Number of particles	100
$iter_{max}$	Number of iterations	250
X_{space}	Define the search space	[1,64]

Example 1: Planar array with $M \times N = 10 \times 10$ elements with the aim to produce a radiation pattern that has a -40 dB SLL and an HPBW of 14.4° with two nulls; one at $(\theta_{n1}, \phi_{n1}) = (30^\circ, 0^\circ)$ and the second one at $(\theta_{n2}, \phi_{n2}) = (-30^\circ, 0^\circ)$ with the maximum radiation pattern directed at $(\theta_d, \phi_d) = (0^\circ, 0^\circ)$. The fitness function of Eq. (5) is the cost function and the array factor of Eq. (4) is used to compute the radiation pattern. The new current values of the elements are illustrated in Table 2.

Table 2. Optimized asymmetric current excitation of 10×10 planar antenna array synthesized using IPSO.

SLL	Null Angle		Null Depth		HPBW		Directivity			
-40 dB ($\phi = 0^\circ$)	$(-30^\circ, 0^\circ)$		-80 dB		14.9 ($\phi = 0^\circ$)		22 dB			
-26 dB ($\phi = 90^\circ$)	$(30^\circ, 0^\circ)$		-80 dB		13.7 ($\phi = 90^\circ$)					
Magnitude of current excitation										
w_{mn}	n1	n2	n3	n4	n5	n6	n7	n8	n9	n10
m1	1	3.14	12.34	7.22	7.68	9.73	4.41	14.47	2.82	1
m2	1	6.78	5.83	20.07	29.10	26.97	18.07	4.20	4.88	2.60
m3	7.02	10.52	20.85	35.11	38.39	30.65	34.17	21.53	11.22	4.17
m4	6.18	17.56	29.61	44.78	51.77	55.74	43.75	32.22	15.52	5.98
m5	7.20	20.64	36.52	53.37	63.50	63.55	54.05	36.30	18.60	7.39
m6	6.58	20.28	37.41	54.48	63.83	63.90	50.32	36.19	15.16	5.43
m7	5.11	17.33	29.56	38.44	52.45	54.50	44.10	31.69	16.91	4.69
m8	5.73	12.04	22.34	32.99	36.50	37.40	31.57	22.77	10.62	5.75
m9	2.38	8.45	4.52	26.47	16.49	20.92	18.42	8.06	4.16	1.18
m10	1	4.26	24.68	14.51	28.19	13.32	10.99	3.04	1.02	1

Table 2 shows the results of current excitation after the array has been optimized using IPSO, where it can be seen that the current magnitude gets higher as we move from the edges to the center.

Figure 3 shows the resulting radiation pattern after optimizing the current excitation at $\phi = 0^\circ$, while Figure 4 shows the resulting radiation pattern at $\phi = 90^\circ$. Since the current being optimized is asymmetric, the radiation pattern of these two planes ($\phi = 0^\circ, \phi = 90^\circ$) may and may not be identical. In this example, they are not.

It can be seen from Figure 3 that the radiation pattern at $\phi = 0^\circ$ achieves the desired SLL of -40 dB and places the nulls at the desired angle with a very good depth of -80 dB, where the HPBW achieved is 14.9° which is 0.4° higher than desired. In the other plane in Figure 4, the maximum SLL near the main beam is -31 dB, while the SLL at the edges is about -26.2 dB. This is considered a good result, since the fitness function does not evaluate the fitness of the algorithm at this plane, it yields an HPBW of 13.7° which is better than desired by 0.8° .

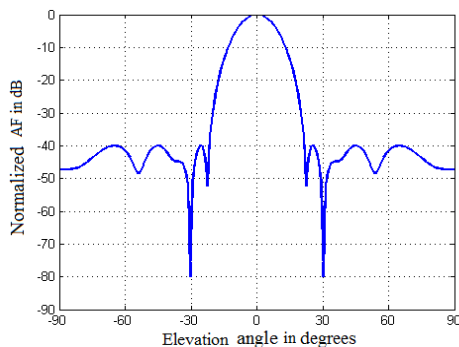


Figure 3. Synthesized radiation pattern of a 10×10 planar array antenna using IPSO for asymmetric current excitation at $\phi = 0^\circ$.

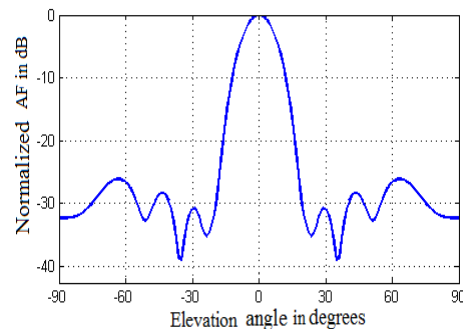


Figure 4. Synthesized radiation pattern of a 10×10 planar array antenna using IPSO for asymmetric current excitation at $\phi = 90^\circ$.

The average fitness (mean value of p_{best} in red line) and the fitness (g_{best} in blue line) of each iteration for IPSO are illustrated in Figure 5, which shows that the IPSO algorithm reaches its steady state and converges after 60 iterations.

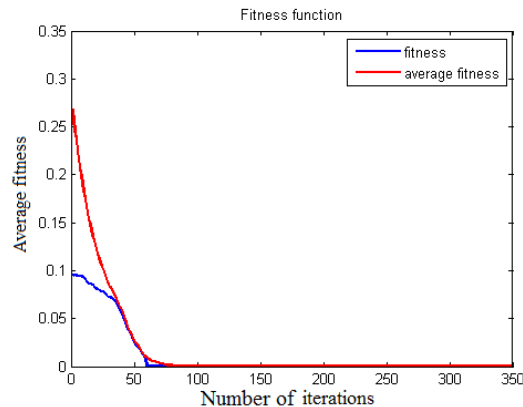


Figure 5. The average fitness of the global best value for each iteration using IPSO.

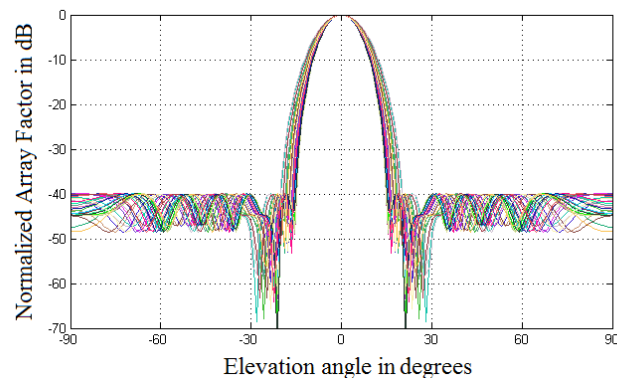


Figure 6. 2-D normalized AF of a 10×10 planar array antenna using IPSO for asymmetric current excitation at different angles ϕ .

Figure 6 exhibits the 2-D normalized radiation pattern of the optimized planar array at different angles ϕ , where it is obvious that the SLL being suppressed as desired.

4.2 Thinned Planar Array Design

The aim here is to find the best distribution of the active (on) elements to maintain the desired requirements of the filled array in example 1 with tapered current excitation. Here, the algorithm randomly turns on and off the elements, then evaluates the fitness of the resulting array factor. The goal is to maintain the same array factor characteristics of the filled array with tapered current.

Example 2: A 10×10 planar array with optimized current excitation using IPSO and uniform spacing with $d=0.5\lambda$ is considered to be thinned. The current values are in Table 2 and the desired array factor is shown in Figure 3; a maximum SLL of -40 dB with an HPBW of 14.9° at $(\phi = 0^\circ)$, a maximum SLL of -26.2 dB with an HPBW of 13.7° at $(\phi = 90^\circ)$ and two nulls at $(\theta, \phi) = (-30^\circ, 0^\circ)$, $(\theta, \phi) = (30^\circ, 0^\circ)$. The fitness function of Eq. (5) is the cost function, and the array factor of Eq. (4) is used to compute the radiation pattern.

The new distribution of the active elements in the 10×10 thinned planar array is illustrated in Table 3. Figure 7 shows the resulting radiation pattern of the thinned array at $\phi = 0^\circ$, while Figure 8 shows the resulting radiation pattern at $\phi = 90^\circ$.

After thinning, 10 elements have been turned off, and the resulting radiation pattern has minor changes far from the main lobe, which is considered a very good result. In this paper, thinning is

limited to only 10% of the array elements, because the thinned array elements have a different (tapered) current excitation to achieve the desired array factor. Therefore, eliminating any elements will have greater impact on the array factor than the usual thinning which is applied to uniform current arrays. However, it is shown that when using uniform current distribution, 42% of the elements can be turned off [4].

In the given example, turning off more than 10 elements will cause undesired characteristics in the resulting array factor, such as higher SLL or higher HPBW. So, the maximum number of elements that can be turned off while maintaining the desired array factor is 10 elements.

Table 3. Thinned array of a 10×10 planar array with optimized current excitation synthesized using IPSO.

SLL		Null Angle		Null Depth		HPBW		Directivity		
-35 dB ($\phi = 0^\circ$)		$(-30^\circ, 0^\circ)$		-64 dB		14.9° ($\phi = 0^\circ$)		21.9 dB		
-27 dB ($\phi = 90^\circ$)		$(30^\circ, 0^\circ)$		-64 dB		14° ($\phi = 90^\circ$)				
Magnitude of current excitation										
w_{mn}	n1	n2	n3	n4	n5	n6	n7	n8	n9	n10
m1	1	3.14	12.34	7.22	7.68	9.73	4.41	0	2.82	1
m2	0	0	5.83	20.07	29.10	26.97	18.07	4.20	0	2.60
m3	7.02	10.52	20.85	35.11	38.39	30.65	0	21.53	11.22	4.17
m4	0	17.56	29.61	44.78	51.77	55.74	43.75	32.22	15.52	5.98
m5	7.20	20.64	36.52	53.37	63.50	63.55	54.05	36.30	18.60	7.39
m6	6.58	20.28	37.41	54.48	63.83	63.90	50.32	36.19	15.16	5.43
m7	5.11	17.33	29.56	38.44	52.45	54.50	44.10	31.69	16.91	4.69
m8	0	12.04	22.34	32.99	36.50	37.40	31.57	22.77	10.62	5.75
m9	2.38	8.45	4.52	26.47	0	20.92	18.42	8.06	4.16	1.18
m10	1	4.26	24.68	14.51	28.19	0	10.99	0	1.02	1

It can be seen from Figure 7 that the radiation pattern at $\phi = 0^\circ$ achieves the desired SLL of -40 dB near the main lobe, where at the edges the SLL increases to -35 dB, placing the nulls at the desired angles with a good depth of -64 dB, where the HPBW achieved is as desired. In the other plane as shown in Figure 8, the maximum SLL near the main beam is -32 dB, while the SLL at the edges is about -27 dB which is better than desired. This is considered a good result, since the fitness function does not evaluate the fitness of the algorithm at this plane.

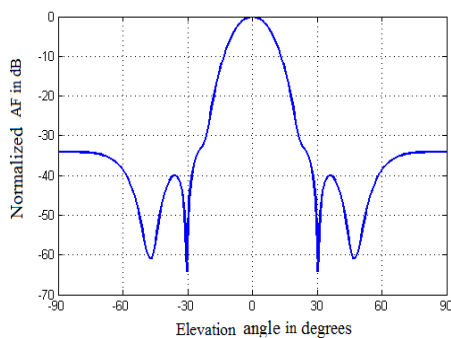


Figure 7. Synthesized radiation pattern of a 10×10 tapered thinned planar array antenna using IPSO at $\phi = 0^\circ$.

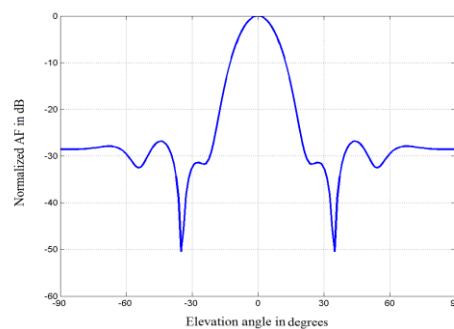


Figure 8. Synthesized radiation pattern of a 10×10 tapered thinned planar array antenna using IPSO at $\phi = 90^\circ$.

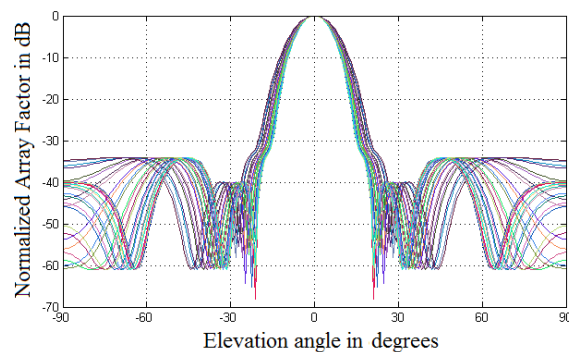


Figure 9. 2-D normalized AF of a 10×10 tapered thinned planar array antenna using IPSO for asymmetric current excitation at different angles ϕ .

Figure 9 exhibits the 2-D normalized radiation pattern of the optimized thinned planar array at different angles ϕ , it is obvious that the SLL are suppressed as desired.

5. CONCLUSIONS

In this paper, IPSO has been successfully used to optimize a planar array antenna. The efficiency of the algorithm especially in multi-objective optimization has been shown. By adjusting the current excitation, the algorithm was able to find the optimum value for the desired properties with fast convergence. The designed array has achieved the desired requirements for the filled array. Moreover, these requirements have been maintained for the thinned planar array. Thinning was limited to only 10% of the array elements, because the thinned array elements have different (tapered) current excitation to achieve the desired array factor. Therefore, eliminating any element will have greater impact on the array factor than usual thinning which is applied to uniform current arrays.

As the examples have been illustrated, turning off more than 10 elements will cause undesired characteristics in the resulting array factor, such as higher SLL or higher HPBW. So, the maximum number of elements that can be turned off while maintaining the desired array factor amount to 10 elements.

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ملخص البحث:

تمّ في هذا البحث، إتباع نهج جديد للوصول إلى الحلّ الأمثل للمصفوفة الهوائية السطحية باستخدام توزيع Chebyshev وتحسين الاستفادة المثلى لسرب الجسيمات. استخدم تيار الإثارة لكل عنصر عامل استفادة مثلى بهدف تقليل مستوى الإشعاع من عرض شعاع منتصف القدرة مع الكبح المسبق للإشعاع في اتجاهات معينة. و الجدير بالإشارة أن توزيع Chebyshev يستخدم لتحديد فضاء البحث لسرب الجسيمات المحسّن. و قد تم ترقيق المصفوفة نفسها لإيجاد أفضل توزيع للعناصر النشطة، أو العاملة؛ من أجل الحصول على المتطلبات المطلوبة.