# Full-rate Space-time Block Code for Four Transmit Antennas with Linear Decoding 

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(Received: 3-Feb.-2020, Revised: 22-Apr.-2020, Accepted: 7-May-2020)


#### Abstract

This paper proposes a new full-rate space-time block code for MIMO systems at four transmit antennas, which aims at minimizing the bit error rate with low-complexity decoding using ZF and MMSE linear detection techniques in Rayleigh fading channels. The purpose of this code is to optimize the space-time resources offered by MIMO systems and to ensure optimal exploitation of spectral resources. The idea is to take advantage of the direct sequence spread spectrum (DSSS) technique using the orthogonal codes of Walsh-Hadamard in order to ensure the orthogonality between all the symbol vectors to be transmitted and to construct an orthogonal and full-rate STBC code. BER performance versus Eb/No of the proposed STBC code is evaluated for MIMO $4 \times 2$, $4 \times 4$ and MISO $4 \times 1$ configurations in single-user detection mode, using 16QAM modulation, on Rayleigh's MIMO channels assumed to be quasi-static and not frequency selective, compared with those of the orthogonal STBC of Tarokh of $1 / 2$ rate and the full-rate quasi-orthogonal STBC of Jafarkhani. The results of the simulations show that the proposed STBC code significantly improves BER performance while allowing a higher transmission rate with spectral efficiency of 4 bits $/ s / \mathrm{Hz}$ and simple linear decoding using ZF or MMSE techniques.


## KEYWORDS

MIMO system, STBC, Full-rate, BER, Zero forcing, MMSE.

## 1. INTRODUCTION

Space-time block coding is a very popular diversity approach, used to combat channel fading and minimize bit error rates [1]-[2]. Thanks to this diversity approach, a maximum diversity of $N_{T} \times N_{R}$ equal to the number of independent paths available can be obtained. Orthogonal space-time block codes (OSTBC) have been designed to obtain the maximum diversity order for a given number of transmit and receive antennas using a simple linear decoding algorithm at reception [3]. This has made these codes the most dominant forms of space-time codes, adopted in several wireless communication standards [4].
For an optimal exploitation of spectral resources, it is interesting to have an STBC code with unit rate. The code rate $R_{\text {STBC }}$ is defined as the ratio between the number of transmitted symbols $K$ per code word and the number of symbol durations $T$ during which these symbols are transmitted $R=K / T$. The best STBC codes in the literature are those with a unit rate and achieve maximum diversity for a given number of transmit antennas. Alamouti's complex STBC code using two transmit antennas, proposed by S. Alamouti in 1998, is the only full-rate orthogonal complex code [5]. This code allows maximum diversity using a single antenna for reception. Alamouti code then is the only STBC code making it possible to achieve these two characteristics characterizing the optimal STBC code, with acceptable BER performance at reception [5]. Therefore, Alamouti code is a very special code and one of the most widely adopted codes.
Increasing the number of both transmit and receive antennas improves system performance and reduces bit error rate [6]-[7]. In this setting, in 1999, V. Tarokh et al. had the initial idea to generalize the concept of Alamouti to a number of transmit antennas more than two by reducing the code rate ( $R_{\text {STBC }}<1$ ) and by leaving diversity and orthogonality unchanged [8]. Consequently, these lower rate codes alter the transmission rate and degrade the spectral efficiency of the MIMO system [9]. Full-rate quasi-orthogonal codes (QOSTBCs) have been proposed to achieve total transmission rate for wireless systems with more than two transmit antennas, at the cost of losing some of the diversity and increasing the decoding complexity. However, these codes affect performance in error rates at low and

[^0]high SNR, which limits their use [10]. Many unit rate codes have been developed, including codes based on Alamouti code with switching between antenna groups, linear dispersion codes and DAST diagonal algebraic codes, allowing maximum diversity. However, some of them present higher complexity in coding and/or decoding or even limited BER performance. Full-rate linear dispersion algebraic codes have been the subject of several studies until today; however, the disadvantage of this type of STBC codes lies in the difficulty of optimizing the dispersion matrices (they are dependent on the constellation) in order to maximize diversity, as well as high complexity in coding. This paper proposes a new full-rate space-time block code for MIMO systems for four transmit antennas, which aims at minimizing the bit error rate with low-complexity decoding using ZF linear detection technique in Rayleigh fading channels. The purpose of this code is to optimize the space-time resources offered by MIMO systems and therefore allow optimal exploitation of spectral resources. The paper is organized as follows: in Section 2, the system model of STBC-MIMO is described. Next, main existing STBC codes for MIMO systems of four transmit antennas are presented in Section 3. In Section 4, the proposed STBC code for four transmit antennas is presented. In Section 5, the simulation methodology is discussed; then, results and analysis are presented. Finally, Section 6 concludes this paper.

## 2. System Model

The idea behind MIMO systems based on space-time block codes (STBC) is to transmit data in such a way as to guarantee great diversity, while allowing a simple decoding process. The STBC code consists of blocks. Each block is coded according to precise rules and independently of the other blocks. The idea is to send each block of symbols to the $N_{T}$ transmit antennas. A decoding error occurring in one block does not jeopardize other blocks.


Figure 1. STBC-MIMO system.
A space-time block code is generally represented by an $N_{T} \times T$ matrix, where each line represents a transmit antenna and each column represents a block of symbols of block duration:

$$
C=\left(\begin{array}{cccc}
s_{11} & s_{12} & \cdots & s_{1 T}  \tag{1}\\
s_{21} & s_{22} & \cdots & s_{2 T} \\
\vdots & \vdots & \vdots & \vdots \\
s_{N_{T} 1} & s_{N_{T} 2} & \cdots & s_{N_{T} T}
\end{array}\right)
$$

where $s_{i j}$ is the coded symbol on the $\mathrm{i}^{\text {th }}$ transmit antenna at the $\mathrm{j}^{\text {th }}$ time slot.

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For a MIMO system with $N_{T}$ transmit antennas and $N_{R}$ receive antennas, the received signal $y_{j}$ at each moment on the $\mathrm{j}^{\text {th }}$ receive antenna is the sum of the symbols derived from the $N_{T}$ transmitted signals:

$$
\begin{equation*}
y_{i}=\sum_{i=1}^{N_{T}} h_{i, j} s_{j}+n_{i} \tag{2}
\end{equation*}
$$

where $h_{i, j}$ is the attenuation and phase shift (transfer function) of the non-selective frequency channel between the $\mathrm{j}^{\text {th }}$ transmit antenna and $\mathrm{i}^{\text {it }}$ receive antenna and $n_{i}$ is the additive noise. The complex matrix H of the channel can then be written as follows:

$$
H=\left(\begin{array}{cccc}
h_{1,1} & . & . . & h_{1, N_{T}}  \tag{3}\\
. & h_{2,2} & . . & . \\
: & : & : & : \\
h_{N_{R}, 1} & h_{N_{R}, 2} & . . & h_{N_{R}, N_{T}}
\end{array}\right)
$$

The MIMO signal model can be described as:

$$
\begin{equation*}
y=H \cdot C+n \tag{4}
\end{equation*}
$$

where $y$ and $n$ are respectively receive and noise vectors of size $N_{R} \times 1$, H is the channel matrix of size $N_{R} \times N_{T}$ and c is the transmitted vector of size $N_{T} \times 1$.
Each STBC code is characterized by a code rate $\mathrm{R}_{\text {StBc }}$. The code rate $\mathrm{R}_{\text {STBC }}$ is defined as the ratio between the number of transmitted symbols $K$ per code word and the number of symbol durations $T$ during which these symbols are transmitted:

$$
\begin{equation*}
R=K / T \tag{5}
\end{equation*}
$$

The STBC $C$ code is orthogonal if it satisfies the following criteria:

$$
\begin{equation*}
C \cdot C^{H}=A \cdot I_{n} \tag{6}
\end{equation*}
$$

where $C^{H}$ denotes the Hermitian transpose of the code matrix, $A$ is a real coefficient and $I_{n}$ is the identity matrix of size $n \times n(n \in I N)$.

## 3. Main Existing STBC For Four Transmit Antennas

### 3.1 Orthogonal-STBC Codes of Tarokh

Tarokh et al. proposed complex orthogonal space-time block codes in [11] for four transmit antennas. These codes provide maximum diversity; however, they have the disadvantage of having a rate lower than one. The matrices of codes $C_{4}$ and $X_{4}$ of rates $R_{S T B C}=1 / 2$ and $R_{S T B C}=3 / 4$, respectively, for four transmit antennas, are as follows:

$$
\begin{align*}
& C_{4}=\left(\begin{array}{cccccccc}
s_{1} & -s_{2} & -s_{3} & -s_{4} & s_{1}^{*} & -s_{2}^{*} & -s_{3}^{*} & -s_{4}^{*} \\
s_{2} & s_{1} & s_{4} & -s_{3} & s_{2}^{*} & s_{1}^{*} & s_{4}^{*} & -s_{3}^{*} \\
s_{3} & -s_{4} & s_{1} & s_{2} & s_{3}^{*} & -s_{4}^{*} & s_{1}^{*} & s_{2}^{*} \\
s_{4} & s_{3} & -s_{2} & s_{1} & s_{4}^{*} & s_{3}^{*} & -s_{2}^{*} & s_{1}^{*}
\end{array}\right)  \tag{7}\\
& X_{4}=\left(\begin{array}{cccccc}
s_{1} & -s_{2}^{*} & \frac{s_{3}^{*}}{\sqrt{2}} & \frac{s_{3}^{*}}{\sqrt{2}} \\
s_{2} & s_{1}^{*} & \frac{s_{3}^{*}}{\sqrt{2}} & & -\frac{s_{3}^{*}}{\sqrt{2}} \\
\frac{s_{3}}{\sqrt{2}} & \frac{s_{3}}{\sqrt{2}} & \frac{-s_{1}-s_{1}^{*}+s_{2}-s_{2}^{*}}{2} & \frac{s_{2}+s_{2}^{*}+s_{1}-s_{1}^{*}}{2} \\
\frac{s_{3}}{\sqrt{2}} & -\frac{s_{3}}{\sqrt{2}} & \frac{-s_{2}-s_{2}^{*}+s_{1}-s_{1}^{*}}{2} & -\frac{s_{1}+s_{1}^{*}+s_{2}-s_{2}^{*}}{2}
\end{array}\right) \tag{8}
\end{align*}
$$

where $s_{1}, s_{2}, s_{3}$ and $\mathrm{s}_{4}$ are the symbols to be transmitted and (.) ${ }^{*}$ denotes the complex conjugate.
The channel matrices obtained are therefore orthogonal. Consequently, the decoding of the symbols can be done simply with the maximum likelihood technique while reducing the processing complexity.

### 3.2 Quasi-OSTBC Codes

Interesting compromises for MIMO systems at four transmit antennas were proposed by H. Jafarkhani in [12] and Tirkkonen in [13]. The idea is to create quasi-orthogonal complex codes admitting a simplified maximum likelihood decoding (but more complex than the decoding of an orthogonal code).
The matrices of full-rate quasi-orthogonal space-time block codes $C_{J a f}$ and $C_{T i r}$ proposed by Jafarkhani and Tirkkonen, respectively, are as follows:

$$
\begin{gather*}
C_{\text {Jaf }}=\left(\begin{array}{cccc}
s_{1} & -s_{2}^{*} & -s_{3}^{*} & s_{4} \\
s_{2} & s_{1}^{*} & -s_{4}^{*} & -s_{3} \\
s_{3} & -s_{4}^{*} & s_{1}^{*} & -s_{2} \\
s_{4} & s_{3}^{*} & s_{2}^{*} & s_{1}
\end{array}\right)  \tag{9}\\
C_{\text {Tir }}=\left(\begin{array}{cccc}
s_{1} & -s_{2}^{*} & s_{3} & -s_{4}^{*} \\
s_{2} & s_{1}^{*} & s_{4} & s_{3}^{*} \\
s_{3} & -s_{4}^{*} & s_{1} & -s_{2}^{*} \\
s_{4} & s_{3}^{*} & s_{2} & s_{1}^{*}
\end{array}\right) \tag{10}
\end{gather*}
$$

Both full-rate quasi-orthogonal codes of Jafarkhani and Tirkkonen have the same BER performance at low and high SNR in Rayleigh fading channels [14].
The orthogonal STBC developed by Tarokh with a rate of $1 / 2$ and the full-rate quasi-orthogonal STBC of Jafarkhani, for four transmit antennas, are widely used and most adopted in modern and futuristic digital wireless communication systems, due to their lower complexities in coding and decoding. For this reason, they constitute the reference codes for simulating the performance of our proposed STBC.

## 4. Proposed STBC Code

The principle of proposed STBC coding in this research aims at exploiting the technique of spectrum spreading by direct sequences (DSSS) using the orthogonal codes of Walsh-Hadamard, in order to ensure orthogonality between all the symbol vectors to be transmitted and build an orthogonal STBC code at a rate of 1 .
The idea is that each of the four information symbols to be transmitted is multiplied in the time domain by its own Walsh-Hadamard code (correlation product). The four resulting symbols to be sent are applied to a square matrix of order 4 for real space-time coding, where each of these symbols is different from all adjacent symbols and is orthogonal with them, as shown in Table 1. Our design criteria for the proposed code do not depend on the used constellation.
Walsh-Hadamard codes are not very long or PN type for a significant spread spectrum. WalshHadamard codes are adopted in our designed STBC-MIMO system for their orthogonality, ease of generation and simplicity of implementation. They provide optimal performance in the presence of synchronous transmission.
Figure 2 presents the general scheme of the proposed transmit system. It comprises four main subsystems: modulator, serial-parallel converter, Walsh-Hadamard coder and space-time coder in blocks. After mapping operation, data to be transmitted is converted from series into parallel from with four symbols at the output. Then, these four symbols are correlated (multiplication in the time domain) with their different four own Walsh-Hadamard codes of the same length, as shown in Figure 3. We can express this in mathematical form by writing the Walsh-Hadamard spreading code vectors and the information symbol vectors successively in the matrices C and s :

$$
\begin{align*}
& C=\left[\begin{array}{llll}
c_{1} & c_{2} & c_{3} & c_{4}
\end{array}\right]  \tag{11}\\
& s=\left(\begin{array}{llll}
x_{1} & x_{2} & x_{3} & x_{4}
\end{array}\right) \tag{12}
\end{align*}
$$

To carry out the matrix multiplication of the code vectors of $C$ with the matrix $s$ of the information symbol vectors, it will be advantageous to put the matrix $C$ in diagonal form:

$$
C=\operatorname{diag}\left(c_{1} \begin{array}{c}
c_{2} \tag{13}
\end{array} c_{3} \quad c_{4}\right)
$$

The implicit form of multiplication can then be represented by:

$$
S=s . C=\left(\begin{array}{llll}
s_{1} & s_{2} & s_{3} & s_{4} \tag{14}
\end{array}\right)
$$

Finally, the reconverted symbols from parallel form into serial form apply to the space-time block coder for a real space-time coding in accordance with the following square coding matrix $4 \times 4$, where each of these symbols is different from all adjacent symbols.

Table 1. Space-time coding of the proposed $4 \times 4$ code.

|  | Antenna 1 | Antenna 2 | Antenna 3 | Antenna 4 |
| :---: | :---: | :---: | :---: | :---: |
| Time t | $\mathrm{s}_{1}$ | $\mathrm{~s}_{2}$ | $\mathrm{~s}_{3}$ | $\mathrm{~s}_{4}$ |
| Time t+T | $\mathrm{s}_{2}$ | $\mathrm{~s}_{1}$ | $\mathrm{~s}_{4}$ | $\mathrm{~s}_{3}$ |
| Time t+2T | $\mathrm{s}_{3}$ | $\mathrm{~s}_{4}$ | $\mathrm{~s}_{1}$ | $\mathrm{~s}_{2}$ |
| Time t+3T | $\mathrm{s}_{4}$ | $\mathrm{~s}_{3}$ | $\mathrm{~s}_{2}$ | $\mathrm{~s}_{1}$ |

The number of symbols transmitted $K$ per codeword is equal to the number of time slot $T$ necessary to transmit these symbols; $K=T=4$, then, $R_{S T B C}=K / T=1$. The proposed STBC code for four transmit antennas is therefore full-rate and is presented as follows:

$$
C_{S T B C, 4}=\left(\begin{array}{llll}
s_{1} & s_{2} & s_{3} & s_{4}  \tag{15}\\
s_{2} & s_{1} & s_{4} & s_{3} \\
s_{3} & s_{4} & s_{1} & s_{2} \\
s_{4} & s_{3} & s_{2} & s_{1}
\end{array}\right)
$$

with $s=x_{i} \cdot c_{j}$, where $x_{i}$ is the information symbol to be transmitted and $c_{j}$ is the Walsh-Hadamard spreading code specific to the information symbol $x_{i}$. The matrix of the proposed STBC code is symmetrical:

$$
C_{S T B C}=C_{S T B C}^{H}=\left(\begin{array}{llll}
s_{1} & s_{2} & s_{3} & s_{4}  \tag{16}\\
s_{2} & s_{1} & s_{4} & s_{3} \\
s_{3} & s_{4} & s_{1} & s_{2} \\
s_{4} & s_{3} & s_{2} & s_{1}
\end{array}\right)
$$

Each symbol $s$ of the proposed STBC code is a correlation product of a useful symbol with its own Walsh-Hadamard code. The four signals obtained from the four correlation products are orthogonal to each other. The proposed STBC coding scheme is shown in Figure 3.


Figure 2. Block diagram of the proposed STBC-MIMO system in transmission.

As mentioned above, the design criteria for the proposed STBC code do not depend on the constellation used and the useful symbol $x_{i}$ can be from a real or complex constellation.


Figure 3. Principle of the proposed STBC coding.
Walsh-Hadamard codes of a length n are constructed from a Hadamard matrix of an order n . An example of the 8 -bit Hadamard codes used in this study is shown in Figure 4.
The length of the 8 -bit Walsh-Hadamard orthogonal codes was adopted in our proposed STBC code for simulations so that they ensure the orthogonality of the STBC code, and provide a better compromise between the used frequency band and the processing complexity especially at decoding. W-H sequences of length greater than 8 can be used ( 16 bit, 64 bit...).


Figure 4. 8-bit Walsh-Hadamard codes from Hadamard matrix of order 8.
Due to this orthogonality property, cross-correlation between the four symbols $s_{1}, s_{2}, s_{3}$ and $s_{4}$ transmitted simultaneously every four time slots is zero due to perfect synchronization of their transmission.
$\mathrm{s}_{1}(\mathrm{n}), \mathrm{s}_{2}(\mathrm{n}), \mathrm{s}_{3}(\mathrm{n})$ and $\mathrm{s}_{4}(\mathrm{n})$ are orthogonal to each other over the interval $\left[\mathrm{n}_{1}, \mathrm{n}_{8}\right]$ and their scalar product is zero :

$$
\begin{align*}
& \sum_{n=n_{1}}^{n_{8}} s_{1}(n) s_{2}(n)=0  \tag{17}\\
& \sum_{n=n_{1}}^{n_{8}} s_{1}(n) s_{3}(n)=0  \tag{18}\\
& \sum_{n=n_{1}}^{n_{8}} s_{1}(n) s_{4}(n)=0  \tag{19}\\
& \sum_{n=n_{1}}^{n_{8}} s_{2}(n) s_{3}(n)=0  \tag{20}\\
& \sum_{n=n_{1}}^{n_{8}} s_{2}(n) s_{4}(n)=0  \tag{21}\\
& \sum_{n=n_{1}}^{n_{8}} s_{3}(n) s_{4}(n)=0 \tag{22}
\end{align*}
$$

Similarly, all the columns of the code matrix are orthogonal to each other:

$$
\begin{align*}
& \sum_{n=n_{1}}^{n_{8}}\left(s_{1}(n) s_{2}(n)+s_{2}(n) s_{1}(n)+s_{3}(n) s_{4}(n)+s_{4}(n) s_{3}(n)\right)=0  \tag{23}\\
& \sum_{n=n_{1}}^{n_{8}}\left(s_{1}(n) s_{3}(n)+s_{2}(n) s_{4}(n)+s_{3}(n) s_{1}(n)+s_{4}(n) s_{2}(n)\right)=0  \tag{24}\\
& \sum_{n=n_{1}}^{n_{8}}\left(s_{1}(n) s_{4}(n)+s_{2}(n) s_{3}(n)+s_{3}(n) s_{2}(n)+s_{4}(n) s_{1}(n)\right)=0  \tag{25}\\
& \sum_{n=n_{1}}^{n_{8}}\left(s_{2}(n) s_{3}(n)+s_{1}(n) s_{4}(n)+s_{4}(n) s_{1}(n)+s_{3}(n) s_{2}(n)\right)=0  \tag{26}\\
& \sum_{n=n_{1}}^{n_{8}}\left(s_{2}(n) s_{4}(n)+s_{1}(n) s_{3}(n)+s_{4}(n) s_{2}(n)+s_{3}(n) s_{1}(n)\right)=0  \tag{27}\\
& \sum_{n=n_{1}}^{n_{8}}\left(s_{3}(n) s_{4}(n)+s_{4}(n) s_{3}(n)+s_{1}(n) s_{2}(n)+s_{2}(n) s_{1}(n)\right)=0 \tag{28}
\end{align*}
$$

Consequently, the STBC $4 \times 4$ code achieves the following orthogonality criterion:

$$
C_{\text {STBC }} \cdot C_{S T B C}^{H}=\left(\left|s_{1}\right|^{2}+\left|s_{2}\right|^{2}+\left|s_{3}\right|^{2}+\left|s_{4}\right|^{2}\right) \cdot\left(\begin{array}{llll}
1 & 0 & 0 & 0  \tag{29}\\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right)
$$

where $\mathrm{C}^{\mathrm{H}}$ denotes the transposed matrix of the code. The code is then orthogonal.
The orthogonality property of the proposed STBC code allows spatial orthogonality between all the symbol vectors to be transmitted. The proposed orthogonal STBC code also provides inter-channel orthogonality between different transmit antennas. Similarly, the shape of the transmitted symbols (resulting from DSSS) allows a higher resistivity of the signal to interference in the channel and a more robust signal on reception.

The proposed system at the reception performs the inverse operation of the transmit system, as shown in Figure 5, by using one of the linear detection techniques ZF or MMSE.
The equivalent channel matrices obtained $H_{4 \times 1}, H_{4 \times 2}$ and $H_{4 \times 4}$ respectively of the proposed STBC code in MISO $4 \times 1$, MIMO $4 \times 2$ and MIMO $4 \times 4$ configurations, for four time slots, are represented below:

$$
H_{4 \times 1}=\left(\begin{array}{llll}
h_{1} & h_{2} & h_{3} & h_{4}  \tag{30}\\
h_{2} & h_{1} & h_{4} & h_{3} \\
h_{3} & h_{4} & h_{1} & h_{2} \\
h_{4} & h_{3} & h_{2} & h_{1}
\end{array}\right)
$$

where $h_{j}$ is the complex sub-channel coefficient between the $\mathrm{j}^{\text {th }}$ transmit antenna and the receive antenna.

$$
\begin{align*}
& H_{4 \times 2}=\left(\begin{array}{llll}
h_{1,1} & h_{1,2} & h_{1,3} & h_{1,4} \\
h_{2,1} & h_{2,2} & h_{2,3} & h_{2,4} \\
h_{1,2} & h_{1,1} & h_{1,4} & h_{1,3} \\
h_{2,2} & h_{2,1} & h_{2,4} & h_{2,3} \\
h_{1,3} & h_{1,4} & h_{1,1} & h_{1,2} \\
h_{2,3} & h_{2,4} & h_{2,1} & h_{2,2} \\
h_{1,4} & h_{1,3} & h_{1,2} & h_{1,1} \\
h_{2,4} & h_{2,3} & h_{2,2} & h_{2,1}
\end{array}\right)  \tag{31}\\
& H_{4 \times 4}=\left(\begin{array}{llll}
h_{1,1} & h_{1,2} & h_{1,3} & h_{1,4} \\
h_{2,1} & h_{2,2} & h_{2,3} & h_{2,4} \\
h_{3,1} & h_{3,2} & h_{3,3} & h_{3,4} \\
h_{4,1} & h_{4,2} & h_{4,3} & h_{4,4} \\
h_{1,2} & h_{1,1} & h_{1,4} & h_{1,3} \\
h_{2,2} & h_{2,1} & h_{2,4} & h_{2,3} \\
h_{3,2} & h_{3,1} & h_{3,4} & h_{3,3} \\
h_{4,2} & h_{4,1} & h_{4,4} & h_{4,3} \\
h_{1,3} & h_{1,4} & h_{1,1} & h_{1,2} \\
h_{2,3} & h_{2,4} & h_{2,1} & h_{2,2} \\
h_{3,3} & h_{3,4} & h_{3,1} & h_{3,2} \\
h_{4,3} & h_{4,4} & h_{4,1} & h_{4,2} \\
h_{1,4} & h_{1,3} & h_{1,2} & h_{1,1} \\
h_{2,4} & h_{2,3} & h_{2,2} & h_{2,1} \\
h_{3,4} & h_{3,3} & h_{3,2} & h_{3,1} \\
h_{4,4} & h_{4,3} & h_{4,2} & h_{4,1}
\end{array}\right) \tag{32}
\end{align*}
$$

where $h_{i, j}$ is the complex coefficient of sub-channel between the $\mathrm{j}^{\text {th }}$ transmit antenna and the $\mathrm{i}^{\text {it }}$ receive antenna. Assuming that the elements $h_{i, j}$ of the matrix H are independent and identically distributed, and then the columns of the matrix H are independent, the detection of the transmitted symbols from the received vector can be carried out simply by using (ZF or MMSE) linear detection techniques. These two techniques ZF and MMSE consist in applying to the received vector $y$; respectively, the equalization matrices $\mathrm{W}_{\mathrm{ZF}}$ and $\mathrm{W}_{\text {MMSE }}$ are as follows:

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$$
\begin{align*}
& W_{Z F}=\left(H^{H} H\right)^{-1} H^{H}  \tag{33}\\
& W_{M M S E}=\left(H^{H} H+\sigma^{2} I\right)^{-1} H^{H} \tag{34}
\end{align*}
$$

where $(.)^{\mathrm{H}}$ denotes the Hermitian transpose operation and $\sigma^{2}$ is the statistical information of the noise. The estimate of the transmitted signal vector is then given by:

$$
\begin{align*}
& \widehat{s}_{Z F}=W_{Z F} \cdot y  \tag{35}\\
& \widehat{s}_{\text {MASE }}=W_{\text {MMSE }} \cdot y \tag{36}
\end{align*}
$$

Thereafter, the information symbols can be recovered, after STBC decoding, by multiplying each by its own Walsh-Hadamard code (correlation product), as shown in Figure 6.


Figure 5. Proposed reception scheme.


Figure 6. Principle of the proposed STBC decoding.

## 5. RESULTS AND DISCUSSION

The performance of the proposed code is evaluated in the bit error rate (BER) versus signal to noise ratio per bit ( $\mathrm{Eb} / \mathrm{No}$ ) with 16QAM modulation in MATLAB using the quasi-static Rayleigh channel model. The MIMO channels are assumed to be frequency non-selective and not correlated.

The four Walsh-Hadamard codes used for each four symbols to transmit in the simulated proposed
 and $\left[\begin{array}{llllll}1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1\end{array}\right]$, which are shown in Figure 4. Detection is in single-user mode and the channel is assumed perfectly estimated.
Figure 7 and Figure 8 show BER performances obtained versus the signal to noise ratio per bit ( $\mathrm{Eb} / \mathrm{No}$ ) of the proposed STBC code for four transmit antennas with $N_{R}=1,2$ and 4 respectively using ZF and MMSE decoding, compared with the Tarokh orthogonal code $C_{4}$ of $1 / 2$ rate with $N_{R}=1$ and the unit rate quasi-orthogonal code $C_{J a f}$ with $N_{R}=1$, using 16 QAM modulation. It is clear from both figures that the proposed STBC code using ZF or MMSE decoding offers much better BER performance compared to the Tarokh orthogonal code $C_{4}$ of $1 / 2$ rate and the full-rate quasi-orthogonal code of Jafarkhani $C_{J a f}$ using the same modulation 16QAM. Indeed, the proposed STBC code with $N_{R}$ $=4$ has better performance, then comes the proposed code with $N_{R}=2$, afterwards the proposed code with $N_{R}=1$, then the orthogonal code of Tarokh $C_{4}$ and finally the quasi-orthogonal code of Jafarkhani $C_{J a f}$. We also note the significant difference between the BER curves of the proposed code in the MISO system having a theoretical diversity of 4 and the MIMO systems having the higher theoretical diversities of 8 and 16. The obtained results of the performances of the MIMO and MISO systems with the proposed STBC code confirm the theoretical performances in diversity and in decoding based on linear processing ZF and MMSE. In fact, the more the number of receive antennas increases, the lower the curve of the bit error rate becomes.
As shown in Figure 7, the proposed STBC code using ZF decoding with $N_{R}=1$ presents a slight reduction in the error rate in very low signal-to-noise ratios ( $\mathrm{Eb} / \mathrm{No} \leq 5 \mathrm{~dB}$ ) and a noticeable reduction in the ratios $5 \mathrm{db}<\mathrm{Eb} / \mathrm{No}<18 \mathrm{~dB}$; afterwards, its BER curve becomes above that of the OSTBC code $C_{4}$ until $\mathrm{Eb} / \mathrm{No}=25 \mathrm{~dB}$. For MIMO systems with $N_{R}=2$ and 4, the proposed STBC code using ZF decoding presents a significant reduction in the error rate at low and high signal-to-noise ratios compared to the Tarokh orthogonal code and the quasi-orthogonal code of Jafarkhani, as shown in Figure 7. In the case of MMSE decoding, the proposed code with $N_{R}=1$ presents a noticeable reduction in the error rate at low and high signal-to-noise ratios until $\mathrm{Eb} / \mathrm{No}=24 \mathrm{~dB}$ compared to the Tarokh code and a significant reduction in the error rate at low and high signal-to-noise ratios compared to the quasi-orthogonal code of Jafarkhani, as shown in Figure 8. For MIMO systems with $N_{R}=2$ and 4, the proposed STBC code using MMSE decoding presents a significant reduction in the error rate at low and high signal-to-noise ratios compared to the Tarokh orthogonal code and the quasiorthogonal code of Jafarkhani, as shown in Figure 8.


Figure 7. BER performance with 16QAM modulation for ZF decoding.


Figure 8. BER performance with 16 QAM modulation for MMSE decoding.
Figure 9 and Figure 10 show the BER performances of the STBC codes for a spectral efficiency $\eta=4$ bit/s/Hz using ZF and MMSE decoding, respectively. We note that the best performances are obtained by the systems with the proposed STBC code combining the 16QAM modulation. Indeed, the proposed STBC code with $N_{R}=1$ having a theoretical diversity of 4 and using ZF decoding presents good performances in BER at low and high SNR compared to the quasi-orthogonal code of Jafarkhani and at low and high SNR up to $\mathrm{Eb} / \mathrm{No}=21 \mathrm{~dB}$ compared to the Tarokh orthogonal code, as shown in Figure 9. In fact, the Tarokh orthogonal code $C_{4}$ of $1 / 2$ rate with a theoretical diversity of 4 is associated with the 64 QAM modulation which is less robust than the 16QAM modulation; hence, loss of performance (translation of its BER curve upwards) occurs. For MMSE decoding, the proposed STBC code in MISO and MIMO systems presents good performances in BER at low and high SNR compared to the Tarokh orthogonal code and the quasi-orthogonal code of Jafarkhani, as shown in Figure 10.


Figure 9. BER performance at $\eta=4 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ for ZF decoding.


Figure 10. BER performance at $\eta=4 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ for MMSE decoding.
The proposed STBC code in MISO and MIMO systems in case of MMSE decoding provides much better BER performance than in the case of ZF.

## 6. CONCLUSIONS

This paper proposes a new full-rate space-time block code for MIMO systems with four transmit antennas, which aims at minimizing the bit error rate with low-complexity decoding using ZF and MMSE linear detection techniques in Rayleigh fading channels. The simulation results show that the proposed STBC code significantly improves BER performance while allowing simple linear decoding using ZF or MMSE techniques and higher transmission rate with a spectral efficiency of $4 \mathrm{bits} / \mathrm{s} / \mathrm{Hz}$, in a Rayleigh channel assumed to be quasi-static, not frequency selective and spatially uncorrelated. This proposed code allows an optimal exploitation of space-time resources offered by MIMO wireless systems based on space-time coding and therefore, an optimal exploitation of spectral resources.

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 (16 QAM)


 الكامل.


 تقنيتي الكشف (ZF و MMSE).

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