Jordanian Journal of Computers and Information Technology (JJCIT), Vol. 08, No. 01, March 2022.

# AN IMPROVED FRACTIONAL TWO-DIMENSIONAL PRINCIPAL COMPONENT ANALYSIS FOR FACE RECOGNITION

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*(Received: 25-Nov.-2021, Revised: 15-Jan.-2022, Accepted: 26-Jan.-2022)*


## *ABSTRACT*

*Two-dimensional principal component analysis (2DPCA) is a subspace technique used for facial image representation and recognition. Standard 2DPCA may be unable to extract informative features to adequately describe the inherent structural information of the original facial images with the presence of irrelevant variations, such as lighting conditions, facial expressions and so on. To deal with this, an improved fractional two-dimensional principal component analysis (IF2DPCA) is proposed in this paper. It is an extension of fractional 2DPCA (F2DPCA), which was developed based on the concept of fractional covariance matrix (FCM). IF2DPCA employs the same principle as F2DPCA for learning a projective matrix, but further extends the use of fractional transformed 2D images throughout the entire recognition task. As a result, the feature subspace modeled by IF2DPCA maintains the most informative content of the 2D face images and is relatively insensitive to irrelevant variations. Experimental results on three face datasets confirm the effectiveness of the suggested IF2DPCA method in facial recognition.*



## *KEYWORDS*

*Face recognition, Feature extraction, Fractional covariance matrix, 2DPCA, F2DPCA.*


## 1. INTRODUCTION

Face recognition has earned much popularity because of its wide applications in the areas of video surveillance, machine learning and pattern recognition. Among the vast approaches introduced over the years [1]–[4], the most attractive ones are those based on subspace learning techniques [5]. A common paradigm in subspace-based face recognition is to find a subset of features maintaining the informative content of a training set consisting of facial images from distinct classes to be able to correctly assign a class membership to an unknown facial image with the aid of a classifier. Of the subspace learning techniques, the earliest and most widely used is probably the principal component analysis (PCA) [6]–[9]. The PCA procedure consists of mapping high-dimensional input image vectors into a small set of principal components (eigenfaces), describing the most representative content of the input images. Besides, the construction of the eigenfaces is fundamentally dependent on the predominant eigenvectors of the covariance matrix determined from the training image vectors.

Although PCA-based facial recognition methods have exhibited satisfactory recognition accuracy, their formulation requires a preliminary step that unfolds the 2D training images into 1D vectors, inducing high computational cost and loss of inherent structural characteristics of the facial images. To circumvent the implications of image vectorization, two-dimensional principal component analysis (2DPCA) [10] has been developed, wherein the facial images are treated as matrices instead of vectors. The primary purpose of 2DPCA is to create a projective matrix in which the columns are the leading eigenvectors of the covariance matrix evaluated from the row directions of the training instances. In other words, 2DPCA converts each input image to a much smaller feature matrix. This can offer both low computational cost and preservation of the facial structure, so that 2DPCA performs markedly better in most cases than PCA. Following the success of 2DPCA in the representation and recognition of face images, a number of 2DPCA variants have been suggested to improve its performance. Some of them include the bilateral 2DPCA (B2DPCA) [11], horizontal and vertical 2DPCA-based discriminant analysis (HVDA) [12], two-directional two-dimensional 2DPCA ($(2D)^2$PCA) [13], incremental $(2D)^2$PCA (I$(2D)^2$PCA) [14], block-wise $(2D)^2$PCA (B$(2D)^2$PCA) [15] and sequential row-column 2DPCA (RC2DPCA) [16]. The key concept underlying these methods is the projection of face images

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onto two (row-wise and column-wise) projection matrices, simultaneously. While this indeed correlates the row-column information and produces far fewer coefficient features than 2DPCA, it generally yields only a slight improvement in recognition accuracy.

Other efforts have concentrated on the adoption of alternative reconstruction error criteria instead of the L2-norm employed in 2DPCA. The representative ones are L1-norm 2DPCA [17], Lp-norm 2DPCA [18], F-norm 2DPCA [19], nuclear-norm 2DPCA [20], R1-norm [21] and Angle-2DPCA [22]. One major advantage of such type of methods is that they perform quite well in image compression. Nonetheless, their solution relies on iteratively evaluating the projective matrices and as such reducing the flexibility of facial recognition. On the other hand, methods like in [23] and [24] extend classical 2DPCA to class-wise 2DPCA (CW2DPCA) to increase the recognition performance. Instead of establishing a holistic projection matrix from the entire training dataset, CW2DPCA builds multiple projective matrices according to the number of classes that constitute the training dataset. However, this results in a longer computational time when dealing with a large training dataset.

Another approach based on the theory of the fractional covariance matrix (FCM) has been introduced to improve the recognition performance of PCA and 2DPCA. The original idea was presented by Gao et al. [25], who replaced the typical covariance matrix in PCA and 2DPCA with an FCM computed from the fractional transformed training images. The new versions of PCA and 2DPCA are named fractional PCA (FPCA) and fractional 2DPCA (F2DPCA), respectively. This approach has three interesting properties. First, adequate selection of the fractional-order to establish the FCM plays a crucial role in the recognition performance. Second, both FPCA and F2DPCA share the same computational complexity as their classical counterparts. However, third, the features subspace is defined in terms of the dominant eigenvectors of the FCM and original training images, which may deteriorate the spatial quality of the captured information. Within this context, to define better projection while avoiding the curse of dimensionality when dealing with image-as-vector, De Carvalho et al. [26] extended FPCA to fractional eigenfaces (FE), where the feature vectors are generated by applying the eigenface technique to the fractional transformed image vectors. Although FE has demonstrated a recognition advantage over FPCA, it still suffers from drawbacks similar to those of 1DPCA-like methods.

It is important to point out that neither [25] nor [26] have provided a clear justification for why the applications of FCM theory in PCA, eigenfaces and 2DPCA could improve face recognition accuracy. In fact, since in these subspace learning techniques, the learned projection matrix maximizes the overall scatter of the entire training samples, they often retain undesirable variations caused by lighting conditions, shadows, facial expressions and so on [6], [27]. Due to this, it makes sense to employ the FCM theory for scaling down, to some extent, the weights of such variations, thereby mitigating their adverse influence on the performance of face recognition. More details can be found in [28], but regarding the modeling of fractional-order singular value decomposition.

Furthermore, Gao et al. [25] demonstrated the superiority of F2DPCA over 2DPCA in terms of the attained recognition accuracy. Later, F2DPCA is performed in the frequency domain to extract texture information [29]. Despite this endeavor, the main drawback with the F2DPCA model is that the original 2D images are directly involved in the calculation of feature matrices. This could pose a remarkable obstacle towards achieving high recognition performance, as the existence of unwanted variations in the original images may affect the facial appearances and may be substantially high in weight, which can result in a large level of uncertainty in the feature matrices. A practical remedy to this drawback is to extract the feature matrices by means of FCM and fractional transformed 2D images.

In this paper, an extension of F2DPCA, termed improved F2DPCA (IF2DPCA), is proposed to enhance the capability of F2DPCA in face recognition tasks. The proposed IF2DPCA is largely inspired by the fractional transformation in FE and the fractional-order covariance matrix in F2DPCA. In mathematical terms, the IF2DPCA determines a projective matrix, based on the FCM theory, to map the fractional transformed 2D images from the image space to the features subspace, such that the measure of the total scatter in the new subspace is maximal. As a consequence, the feature matrices obtained *via* the IF2DPCA model are not only explained by the structural information of the facial images, but also relatively insensitive to the irrelevant variations, leading to better recognition accuracy than either FE or F2DPCA alone.

In the remainder of this paper, background information and related work are presented in Section 2. Section 3 describes the proposed IF2DPCA method for face recognition. Then, experimental results demonstrating the performance of the IF2DPCA method are reported in Section 4. Finally, conclusions are given in Section 5.

## 2. Background and Related Work

### 2.1 PCA

The computational goal of standard PCA [30] is to identify a set of principal component vectors (eigenvectors) from the covariance matrix of the input dataset such that this set characterizes the variations across the dataset samples, in our case the facial images. Formally, let $\boldsymbol{A} = \{\boldsymbol{A}_i\}_{i=1}^{s}$, $\boldsymbol{A}_i \in \mathbb{R}^{m\times n}$, be a face training dataset with $s$ images and its vectorized version is $\boldsymbol{V} = \{\boldsymbol{a}_i\}_{i=1}^{s}$, $\boldsymbol{a}_i \in \mathbb{R}^{d}$ $(d = mn)$. By defining $\overline{\boldsymbol{a}} = 1/s(\sum_{i=1}^{s} \boldsymbol{a}_i)$ as the total mean of the image vectors, the covariance matrix ($\boldsymbol{C}_{PCA}$) of $\boldsymbol{V}$ can be evaluated as:

$$\boldsymbol{C}_{PCA} = \frac{1}{s}\sum_{i=1}^{s}(\boldsymbol{a}_i - \overline{\boldsymbol{a}})\,(\boldsymbol{a}_i - \overline{\boldsymbol{a}})^T \in \mathbb{R}^{d\times d} = \frac{1}{s}\boldsymbol{H}\boldsymbol{H}^T, \tag{1}$$

where $\boldsymbol{H} = \{\boldsymbol{a}_i - \overline{\boldsymbol{a}}\}_{i=1}^{s} \in \mathbb{R}^{d\times s}$. In practice, applying eigenvalue decomposition to the $\frac{1}{s}\boldsymbol{H}\boldsymbol{H}^T$ matrix is infeasible for typical images. As noted in [31], a simpler alternative solution is determining the eigenvalues and eigenvectors of the matrix $\frac{1}{s}\boldsymbol{H}^T\boldsymbol{H} \in \mathbb{R}^{s\times s}$. Now, suppose that $\frac{1}{s}\boldsymbol{H}^T\boldsymbol{H}$ has the eigenvalue-eigenvector pairs $\{(\lambda_i, \boldsymbol{w}_i) : i = 1, 2, \cdots, s\}$, where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_s$ and that $\boldsymbol{W} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_k] \in \mathbb{R}^{s\times k}$ is formed by keeping the $k$ top eigenvectors. It follows that the PCA projection matrix is $\boldsymbol{W}_{PCA} = \boldsymbol{H}\boldsymbol{W} \in \mathbb{R}^{d\times k}$, in which the column vectors are indeed the first $k$ eigenfaces of $\boldsymbol{V}$. With $\boldsymbol{W}_{PCA}$, the image vectors of $\boldsymbol{V}$ can be simply transformed into a set of reduced training feature vectors written as:

$$\boldsymbol{Y} = \{\boldsymbol{y}_i\}_{i=1}^{s}, \quad \boldsymbol{y}_i = \boldsymbol{W}_{PCA}^T(\boldsymbol{a}_i - \overline{\boldsymbol{a}}) \in \mathbb{R}^{k}. \tag{2}$$

Furthermore, the classification of a test image vector $\boldsymbol{b} \in \mathbb{R}^{d}$ is carried out by comparing its corresponding feature vector $\boldsymbol{x} = \boldsymbol{W}_{PCA}^T(\boldsymbol{b} - \overline{\boldsymbol{a}}) \in \mathbb{R}^{k}$ with the training feature vectors and $\boldsymbol{b}$ ascribes to the class that displays the maximum similarity score. This is usually accomplished through the Euclidean minimum distance procedure.

### 2.2 Fractional PCA

Fractional principal component analysis (FPCA) [25] is a modified version of PCA. From a formulation perspective, the single difference between standard PCA and FPCA is that the former uses a typical covariance matrix, while the latter utilizes the fractional ($r$ −order) covariance matrix, where $0 < r \leq 1$. Note that when $r = 1$, FPCA becomes equivalent to PCA. In this sense, PCA can be viewed as a special case of FPCA.

Under the FPCA assumptions, the FCM ($\boldsymbol{C}_{FPCA}$) of the training dataset $\boldsymbol{V}$ is defined by:

$$\boldsymbol{C}_{FPCA} = \frac{1}{s}\sum_{i=1}^{s}(\boldsymbol{a}_i^r - (\overline{\boldsymbol{a}})^r)\,(\boldsymbol{a}_i^r - (\overline{\boldsymbol{a}})^r)^T \in \mathbb{R}^{d\times d} = \frac{1}{s}(\boldsymbol{H}^r)(\boldsymbol{H}^r)^T, \tag{3}$$

where $\boldsymbol{a}_i^r = (a_{i1}^r, a_{i2}^r, \cdots, a_{id}^r)^T$ and $\boldsymbol{H}^r = \{\boldsymbol{a}_i^r - (\overline{\boldsymbol{a}})^r\}_{i=1}^{s} \in \mathbb{R}^{d\times s}$.

Like standard PCA, FPCA constructs the projection matrix $\boldsymbol{W}_{FPCA} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_k] \in \mathbb{R}^{d\times k}$ by staking the $k$ leading eigenvectors of the $\boldsymbol{C}_{FPCA}$. The training feature vectors $\boldsymbol{Y} = \{\boldsymbol{y}_i\}_{i=1}^{s}$ are subsequently obtained using Eq. (2), but replacing $\boldsymbol{W}_{PCA}$ with $\boldsymbol{W}_{FPCA}$ and likewise for the test samples.

As discussed earlier, with respect to Eq. (1), due to the high dimensionality of the typical images, PCA is unable to directly perform the eigenvalue decomposition to $\boldsymbol{C}_{PCA}$. Unfortunately, this intrinsic limitation is also present in FPCA. One way to address this limitation is by employing the fractional eigenfaces (FE) technique [26], which follows the same procedure as the eigenfaces technique, but assumes fractional transformed image vectors. To be specific, let the column vectors of $\boldsymbol{W} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_k] \in \mathbb{R}^{s\times k}$ be the $k$ leading eigenvectors of the matrix $\frac{1}{s}(\boldsymbol{H}^r)^T(\boldsymbol{H}^r) \in \mathbb{R}^{s\times s}$. In this way, one can obtain $\boldsymbol{W}_{FE} = \boldsymbol{H}^r\boldsymbol{W} \in \mathbb{R}^{d\times k}$ as a projection matrix composed by the first $k$ fractional eigenfaces of the dataset $\boldsymbol{V}$. With $\boldsymbol{W}_{FE}$ in hand, the training feature vectors can be calculated by the following fractional transformation:

$$\boldsymbol{Y} = \{\boldsymbol{y}_i\}_{i=1}^{s}, \quad \boldsymbol{y}_i = \boldsymbol{W}_{FE}^{T}(\boldsymbol{a}_i^r - (\overline{\boldsymbol{a}})^r) \in \mathbb{R}^k. \tag{4}$$

Moreover, $\boldsymbol{x} = \boldsymbol{W}_{FE}^{T}(\boldsymbol{b}^r - (\overline{\boldsymbol{a}})^r) \in \mathbb{R}^k$ is the feature vector of a particular test sample $\boldsymbol{b} \in \mathbb{R}^d$.

Apart from facilitating the application of FPCA, the FE provides a noticeable enhancement in face recognition accuracy. This can mainly be attributed to the projection of the fractional transformed images in place of the original images in FPCA.

### 2.3 2DPCA

In [10], Yang et al. introduced 2DPCA, which, unlike PCA, evaluates the covariance matrix using the 2D images without going through the image-vectorization step. As a result, 2DPCA guarantees appropriate preservation of the facial statistical information with low computational cost, hence benefiting the representation and recognition of the facial images.

In the basic formulation, 2DPCA learns a projection matrix such that the overall scatter of the projected training samples is maximized. More concretely, for the training dataset $\boldsymbol{A} = \{\boldsymbol{A}_i\}_{i=1}^{s}$, $\boldsymbol{A}_i \in \mathbb{R}^{m\times n}$, 2DPCA first determines the image covariance matrix ($\boldsymbol{C}_{2DPCA}$) as:

$$\boldsymbol{C}_{2DPCA} = \frac{1}{s}\sum_{i=1}^{s}(\boldsymbol{A}_i - \overline{\boldsymbol{A}})^T\,(\boldsymbol{A}_i - \overline{\boldsymbol{A}}) \in \mathbb{R}^{n\times n}, \tag{5}$$

where $\overline{\boldsymbol{A}} = 1/s(\sum_{i=1}^{s}\boldsymbol{A}_i)$ denotes the mean image of $\boldsymbol{A}$. After that, the projective matrix $\boldsymbol{W}_{2DPCA} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_k] \in \mathbb{R}^{n\times k}$ is made with the $k$ top orthonormal eigenvectors of $\boldsymbol{C}_{2DPCA}$. It follows that the projection of each training sample $\boldsymbol{A}_i$ onto $\boldsymbol{W}_{2DPCA}$ makes a set of feature matrices according to the number of training samples; that is:

$$\boldsymbol{Y} = \{\boldsymbol{Y}_i\}_{i=1}^{s}, \quad \boldsymbol{Y}_i = \boldsymbol{A}_i\boldsymbol{W}_{2DPCA} \in \mathbb{R}^{m\times k}. \tag{6}$$

For classification, the feature matrix $\boldsymbol{X} = \boldsymbol{B}\boldsymbol{W}_{2DPCA} \in \mathbb{R}^{m\times k}$ of a test image $\boldsymbol{B} \in \mathbb{R}^{m\times n}$ is matched with the training feature matrices and is given a class membership of its nearest neighbor.

### 2.4 Fractional 2DPCA

F2DPCA [25] is similar to 2DPCA, except that the covariance matrix is calculated using the fractional transformed 2D images. This implies that, in addition to the inherited properties from 2DPCA, F2DPCA preserves the facial structural information with less impact from the unwanted variations. In more detail, for the training dataset $\boldsymbol{A}$, the FCM ($\boldsymbol{C}_{F2DPCA}$) is computed as:

$$\boldsymbol{C}_{F2DPCA} = \frac{1}{s}\sum_{i=1}^{s}(\boldsymbol{A}_i^r - (\overline{\boldsymbol{A}})^r)^T\,(\boldsymbol{A}_i^r - (\overline{\boldsymbol{A}})^r) \in \mathbb{R}^{n\times n}. \tag{7}$$

Here, $\boldsymbol{A}_i^r = \left(a_{jl}^r\right)_i$, where $j = 1,2\cdots,m$ and $l = 1,2\cdots,n$. As in 2DPCA, suppose that $\boldsymbol{W}_{F2DPCA} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_k] \in \mathbb{R}^{n\times k}$ is the F2DPCA projection matrix. And along similar lines, the training feature matrices can be produced using Eq. (6) with $\boldsymbol{W}_{2DPCA}$ replaced by $\boldsymbol{W}_{F2DPCA}$ as follows:

$$\boldsymbol{Y} = \{\boldsymbol{Y}_i\}_{i=1}^{s}, \quad \boldsymbol{Y}_i = A_i\boldsymbol{W}_{F2DPCA} \in \mathbb{R}^{m\times k}. \tag{8}$$

This also applies to the test images. So, for a given test image $\boldsymbol{B}$, the feature matrix is obtained as $\boldsymbol{X} = \boldsymbol{B}\boldsymbol{W}_{F2DPCA} \in \mathbb{R}^{m\times k}$.

## 3. IMPROVED F2DPCA

Essentially, the proposed IF2DPCA method can be regarded as an appearance-based modeling problem. In the training phase, IF2DPCA generates compact representations of the facial appearances from a set of fractional transformed 2D images. During the testing phase, given an unknown face image, the face identity can be revealed from the compact representations with the aid of a classifier.

### 3.1 IF2DPCA Formulation

As aforementioned, in the formulation of the F2DPCA model, both the selected eigenvectors of fractional ($r$ −order) covariance matrix and the original training samples have participated in the computation of the feature matrices. In the majority of cases, assuming the original images, there is still a potential for retaining a high level of unwanted information in the projected subspace. Further to this, according to the 2DPCA theory, it is supposed that the projection matrix maximizes the overall scatter of the projected training samples. Arguably, F2DPCA lacks this property. For the sake of dealing with these two concerns, the proposed IF2DPCA method considers the fractional transformed images rather than the original images in the formation of feature matrices. In other words, IF2DPCA and F2DPCA are identical with regard to the definition of the FCM, but differ in the way that they compute the feature matrices. More specifically, IF2DPCA projects the fractional transformed images while F2DPCA projects the raw images.

Let $\boldsymbol{A}^r = \{\boldsymbol{A}_i^r\}_{i=1}^s$, $\boldsymbol{A}_i^r \in \mathbb{R}^{m\times n}$, be the fractional transformed version of the training data $\boldsymbol{A}$ and let $\boldsymbol{W} \in \mathbb{R}^{n\times n}$ be a projection matrix. The projection of each fractional transformed sample into $\boldsymbol{W}$ yields the following projected feature matrices:

$$\boldsymbol{Y}_i = \boldsymbol{A}_i^r \boldsymbol{W} \in \mathbb{R}^{m\times n}, \qquad i = 1, 2, \cdots, s. \tag{9}$$

As indicated in [10], the criterion of the total scatter, $J(\boldsymbol{W})$, can be modeled by means of the trace of the covariance matrix, $\boldsymbol{C}$, of the feature matrices; that is:

$$J(\boldsymbol{W}) = tr(\boldsymbol{C}). \tag{10}$$

In our case, the covariance matrix $\boldsymbol{C}$ is defined by

$$\boldsymbol{C} = \frac{1}{s}\sum_{i=1}^{s}(\boldsymbol{Y}_i - \overline{\boldsymbol{Y}})(\boldsymbol{Y}_i - \overline{\boldsymbol{Y}})^T \in \mathbb{R}^{m\times m} == \frac{1}{s}\sum_{i=1}^{s}[(\boldsymbol{A}_i^r - (\overline{\boldsymbol{A}})^r)\boldsymbol{W}]\,[(\boldsymbol{A}_i^r - (\overline{\boldsymbol{A}})^r)\boldsymbol{W}]^T, \tag{11}$$

where $\overline{\boldsymbol{Y}}$ denotes the mean of the feature matrices. Therefore,

$$tr(\boldsymbol{C}) = \boldsymbol{W}^T\left[\frac{1}{s}\sum_{i=1}^{s}(\boldsymbol{A}_i^r - (\overline{\boldsymbol{A}})^r)^T(\boldsymbol{A}_i^r - (\overline{\boldsymbol{A}})^r)\right]\boldsymbol{W} \in \mathbb{R}^{n\times n} = \boldsymbol{W}^T\boldsymbol{C}_{IF2DPCA}\boldsymbol{W}, \tag{12}$$

where $\boldsymbol{C}_{IF2DPCA} \in \mathbb{R}^{n\times n}$ is the fractional covariance matrix of $\boldsymbol{A}^r$ and it is by default positive semi-definite. Given this, the goal is now to find a set of orthonormal projection vectors, $\boldsymbol{w}_1, \boldsymbol{w}_2 \cdots, \boldsymbol{w}_k$, maximizing $J(\boldsymbol{W})$; that is:

$$\begin{aligned} &\{\boldsymbol{w}_1, \boldsymbol{w}_2 \cdots, \boldsymbol{w}_k\} = \text{argmax} J(\boldsymbol{W}) \\ &s.t.\ \boldsymbol{w}_i^T\boldsymbol{w}_j = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad i,j = 1,2,\cdots,k\,. \end{aligned} \tag{13}$$

This says that these vectors are the $k$ predominant orthonormal eigenvectors of $\boldsymbol{C}_{IF2DPCA}$. Having thus obtained the projection vectors, the IF2DPCA projective matrix can be formed as follows:

$$\boldsymbol{W}_{IF2DPCA} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_k] \in \mathbb{R}^{n\times k}. \tag{14}$$

The $\boldsymbol{W}_{IF2DPCA}$ is then used to transform each fractional transformed image into a features matrix, creating a set of training feature matrices:

$$\boldsymbol{Y} = \{\boldsymbol{Y}_i\}_{i=1}^s, \quad \boldsymbol{Y}_i = \boldsymbol{A}_i^r\boldsymbol{W}_{IF2DPCA} \in \mathbb{R}^{m\times k}. \tag{15}$$

### 3.2 Face Classification

After the training phase, the extracted feature matrices are employed for classification. During testing, upon computing the feature matrix $\boldsymbol{X} = \boldsymbol{B}^r\boldsymbol{W}_{IF2DPCA} \in \mathbb{R}^{m\times k}$ of the fractional transform test image $\boldsymbol{B}^r$, the minimum distance between $\boldsymbol{X}$ and $\boldsymbol{Y} = \{\boldsymbol{Y}_i\}_{i=1}^s$is the evidence that the test image belongs to any of the $c$ classes of the training dataset. More specifically, let $D_j$ denote the minimum distance between $\boldsymbol{X}$ and the feature matrices in the $jth$ class, calculated as follows:

$$D_j = \min_{i\in j}(\|\boldsymbol{X} - \boldsymbol{Y}_i\|_F), \qquad j = 1, 2, \cdots, c. \tag{16}$$

Here, $\|\cdot\|_F$ stands for the standard Frobenius norm. Accordingly, the test image is assigned to the class $j$ for which $D_j$ is the minimum among all the classes.

## 4. EXPERIMENTAL RESULTS

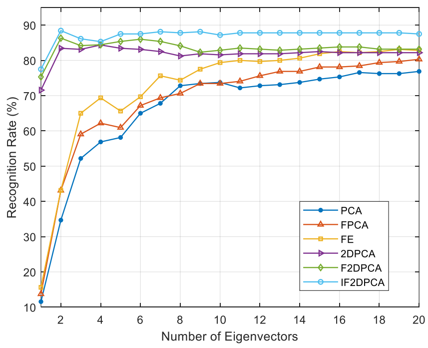
In this section, a set of experiments is presented to confirm the utility of the proposed IF2DPCA model in face recognition. Three public facial datasets (ORL [32], Yale [6] and Georgia Tech [33]) are used to evaluate the recognition performance of IF2DPCA and compare it against PCA [31], FPCA [25], FE [26], 2DPCA [10] and F2DPCA [25]. Throughout experiments, the nearest neighbor classifier is deployed to carry out the classification task. Note that this classifier is based on the Euclidean distance and the Frobenius norm for the 1D and 2D methods, respectively.

## 4.1 Results on ORL

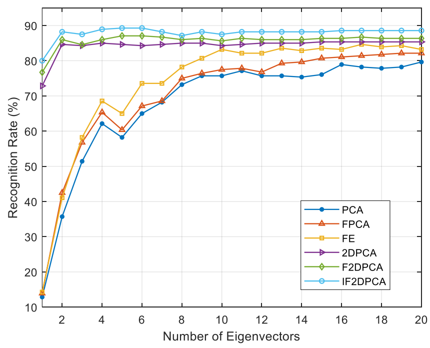
The ORL facial dataset is made up of 40 classes, each with 10 grayscale images. The samples of a distinct class are collected under different lighting conditions, facial expressions, poses and facial details (such as glasses or no glasses). Within this dataset, all images are resized from $112 \times 92$ to $28 \times 23$ pixels [25]. Figure 1 shows the samples of one class in the ORL.

Figure 1. Samples of one class in the ORL dataset.

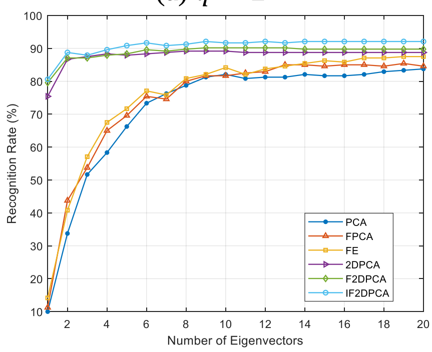
In the experiments, the first $q$ ($q = 2, 3, 4, 5$) images per class are kept to act as the training set and the leftover images compose the testing set. In addition, the value of $r$ is set to 0.01, as this value exhibits the best recognition performance for FPCA-like methods [25] and FE [26] on the ORL dataset. For a certain $q$, the number of eigenvectors ($k$) increases form 1 to 20. Under this setting, the size of the learned projection matrices for PCA, FPCA and FE is $644 \times k$ and for 2DPCA, F2DPCA and IF2DPCA is $23 \times k$. Figure 2 depicts the recognition rates of the six methods. Figure 2 a, b, c and d show the results when $q$ is set to 2, 3, 4 and 5, respectively. As observed, compared with other methods, the recognition rate of the IF2DPCA method is the most dominant when considering the same number of eigenvectors in all cases.



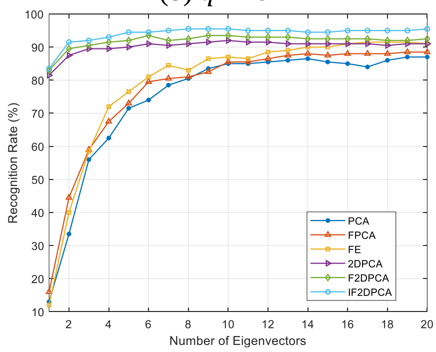

(a) $q = 2$




(b) $q = 3$




(c) $q = 4$




(d) $q = 5$

Figure 2. The recognition rates of the IF2DPCA and competitor methods on the ORL dataset.

The experimental results of each method in terms of maximal recognition rate (MRR) and average recognition rate (ARR) are presented in Table 1. Note that the number of eigenvectors corresponding to the achieved MRR is displayed in parentheses. Based on the results shown in Table 1, the IF2DPCA model consistently produced the MRR at the lowest dimension among the tested methods. For example, with $q = 3$, IF2DPCA achieved MRR of 89.28% ($k = 5$), whereas the MRRs for F2DPCA, 2DPCA, FE, FPCA, and PCA are 87.07% ($k = 5$), 85.35% ($k = 15$), 84.64% ($k = 17$), 82.14% ($k = 19$), and 79.64% ($k = 20$), respectively. From this table, it can be seen again that IF2DPCA outperformed the competitors regarding the ARR with the same number of eigenvectors. In the comparison with the FCM-based methods, IF2DPCA showed improvement over F2DPCA, FE, and FPCA by about 2.5%, 14%, and 17.5%, respectively. This set of results emphasizes the benefits of using the fractional transformed images instead of the original ones in F2DPCA.

## 4.2 Results on Yale

The Yale faces dataset is composed of 165 frontal-view grayscale images representing 15 different classes, where for each class, there are 11 images acquired with various variations in lighting conditions, face expressions and face details. In this group of experiments, the head part of each image is manually cropped and normalized to $40 \times 40$ pixels. The cropped images of one class are shown in Figure 3.

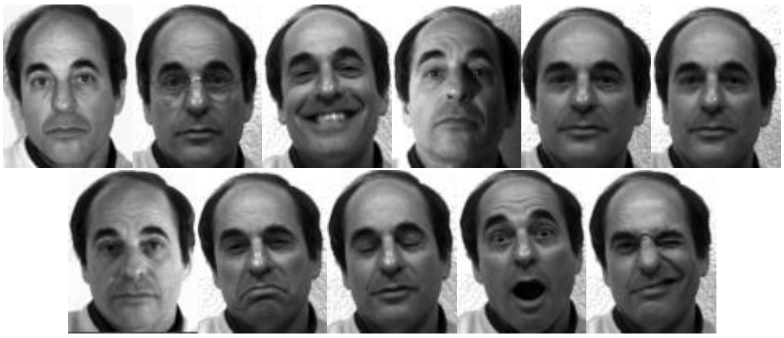
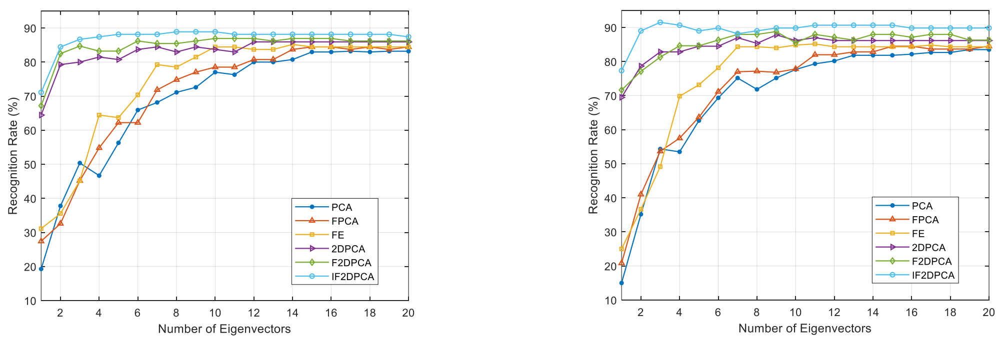


Figure 3. Cropped samples of one person in the Yale dataset.

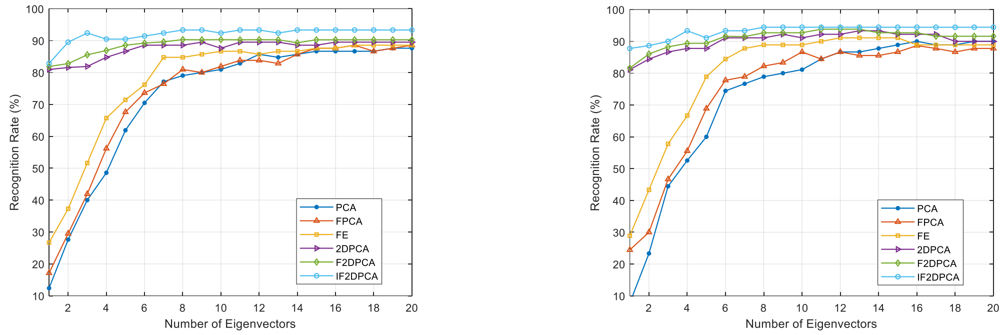
Table 1. The MRR (%) and ARR (%) on the ORL dataset.

| Method | $q = 2$ | | $q = 3$ | | $q = 4$ | | $q = 5$ | |
|---|---|---|---|---|---|---|---|---|
| | MRR ($k$) | ARR | MRR ($k$) | ARR | MRR ($k$) | ARR | MRR ($k$) | ARR |
| PCA | 76.87 (20) | 65.70 | 79.64 (20) | 67.55 | 83.75 (20) | 71.62 | 87.00 (19) | 74.77 |
| FPCA | 80.31 (20) | 68.53 | 82.14 (19) | 70.32 | 85.41 (19) | 74.06 | 88.50 (19) | 77.25 |
| FE | 83.12 (19) | 72.53 | 84.64 (17) | 73.50 | 87.50 (19) | 75.41 | 91.50 (17) | 79.07 |
| 2DPCA | 84.37 (4) | 81.87 | 85.35 (15) | 84.32 | 89.16 (8) | 87.89 | 92.00 (10) | 90.22 |
| F2DPCA | 86.31 (2) | 83.48 | 87.07 (5) | 85.75 | 90.16 (9) | 88.87 | 93.50 (6) | 91.85 |
| IF2DPCA | 88.43 (2) | 87.07 | 89.28 (5) | 87.91 | 92.08 (9) | 90.75 | 95.50 (8) | 94.00 |

In the following experiments, for each class, the first $q$ ($q = 2, 3, 4, 5$) images are chosen to constitute the training set and the rest are used for testing purposes. For the FCM-based methods, the value of $r$ is assigned to be 0.2 [25]. Further, with each $q$, the recognition performance of each method is tested by changing the number of eigenvectors ($k$) from 1 to 20. This implies that the size of the projective matrices for the 1D methods is $1600 \times k$, whereas for the 2D methods, it is $40 \times k$.




(a) $q = 2$ (b) $q = 3$




(c) $q = 4$ (d) $q = 5$

Figure 4. The recognition rates of the IF2DPCA and competitor methods on the Yale dataset.

The obtained recognition rates of the PCA, FPCA, FE, 2DPCA, F2DPCA and IF2DPCA methods are shown in Figure 4. More specifically, Figures 4a, b, c and d display the recognition rates in the cases that $= 2, 3, 4\ and\ 5$, respectively. Table 2 lists the MRRs and ARRs of the six methods.

Table 2. The MRR (%) and ARR (%) on the Yale dataset.

| Method | $q = 2$ | | $q = 3$ | | $q = 4$ | | $q = 5$ | |
|---|---|---|---|---|---|---|---|---|
| | **MRR ($k$)** | **ARR** | **MRR ($k$)** | **ARR** | **MRR ($k$)** | **ARR** | **MRR ($k$)** | **ARR** |
| PCA | 83.18 (17) | 69.03 | 83.50 (19) | 70.46 | 87.61 (19) | 71.95 | 90.00 (16) | 73.07 |
| FPCA | 84.44 (15) | 70.77 | 84.50 (15) | 72.54 | 88.57 (17) | 73.40 | 88.66 (16) | 75.10 |
| FE | 85.18 (14) | 73.88 | 85.16 (11) | 75.70 | 88.57 (17) | 77.30 | 91.11 (12) | 80.67 |
| 2DPCA | 85.92 (12) | 83.07 | 87.83 (9) | 84.58 | 89.52 (9) | 87.60 | 93.33 (13) | 90.11 |
| F2DPCA | 86.92 (10) | 84.82 | 88.80 (9) | 85.44 | 90.31 (8) | 88.86 | 93.82 (11) | 91.09 |
| IF2DPCA | 88.88 (8) | 87.03 | 91.50 (3) | 89.33 | 93.33 (8) | 92.04 | 94.44 (8) | 93.27 |

## 4.3 Results on Georgia Tech

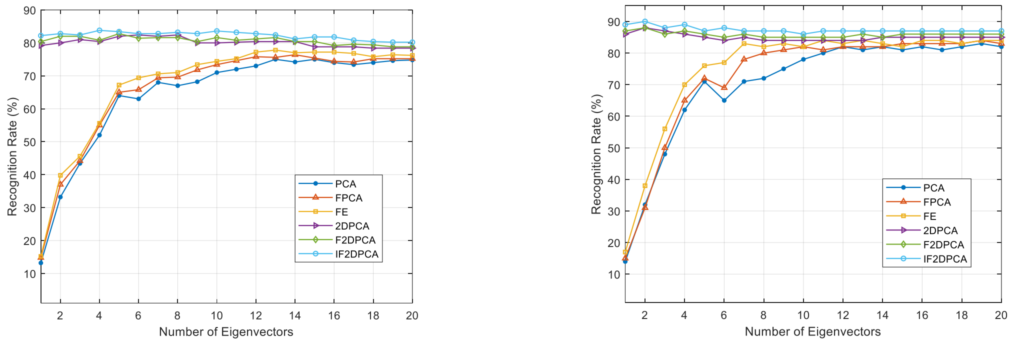
The Georgia Tech face dataset comprises color images of 50 persons, each with 15 facial images taken under different illumination conditions, facial expressions, details and viewpoints. In the experiments, all the images are converted into grayscale and the head part of each image is manually cropped into a size of 50 × 40 pixels. The cropped images of one person are shown in Figure 5.

For evaluation purposes, the first $q = 10$ and 13 images per person are employed to construct the training set and the others served as testing samples. With this dataset, following the suggestion in [25], the value of $r$ is set to 0.01 based on the cumulative contribution rate of the dominant eigenvalues. As before, for each $q$, the number of the eigenvectors ($k$) is increased from 1 to 20. Accordingly, the sizes of the resulting projective matrices for the 1D and 2D methods are $2000 \times k$ and $40 \times k$, respectively.

Figure 5. Cropped images of one person in Georgia Tech face dataset.

The results of the experiments on this dataset are displayed in Figures 6 a and b when $q = 10$ and 13, respectively. As shown, IF2DPCA consistently gives a better recognition rate than any of the five competitor methods. Table 3 reports the achieved MRRs and ARRs by the six methods. As can be seen in the table, with $q = 10$, IF2DPCA reached an MRR of 83.80% at the lowest dimension ($k = 4$) among all the methods. Results from Table 3 also show that IF2DPCA achieved ARRs surpassing those of F2DPCA, 2DPCA, FE, FPCA and PCA by about 1.5%, 2%, 15%, 16% and 18%, respectively.

Furthermore, as shown in the same table, when $q = 13$, the reported MRRs and ARRs of the six methods exhibit the same tendencies as with the previous case. These results further affirm the potential utility of IF2DPCA as an alternative to F2DPCA.




(a) $q = 10$ (b) $q = 13$

Figure 6. The recognition rates of the IF2DPCA and competitor methods on the Georgia Tech dataset.

Table 3. The MRR (%) and ARR (%) on the Georgia Tech dataset.

| | $q = 10$ | | $q = 13$ | |
|---|---|---|---|---|
| **Method** | **MRR ($k$)** | **ARR** | **MRR ($k$)** | **ARR** |
| PCA | 75.00 (13) | 64.15 | 83.00 (19) | 70.20 |
| FPCA | 76.40 (14) | 65.90 | 84.00 (19) | 72.45 |
| FE | 77.80 (13) | 67.45 | 84.00 (11) | 74.95 |
| 2DPCA | 82.40 (6) | 80.12 | 88.00 (2) | 85.00 |
| F2DPCA | 82.80 (5) | 80.74 | 88.00 (2) | 85.85 |
| IF2DPCA | 83.80 (4) | 82.23 | 90.00 (2) | 87.40 |

### 4.4 Discussion

As stated in the introduction, the primary objective of this paper is to utilize the theory of FCM and fractional transformed 2D images in order to develop IF2DPCA as an extension of F2DPCA. The key difference between IF2DPCA and F2DPCA lies in the fact that the former uses fractional transformed images throughout the entire recognition task. Judging by the results of experiments on two facial datasets, the recognition rates are consistently better with the feature matrices extracted by IF2DPCA, so it can be considered as a suitable alternative to F2DPCA. The same remark can also be made when comparing FE with FPCA. In other words, among all the competing methods, IF2DPCA offered the best recognition performance in terms of MRR and ARR, whilst FE outperformed its counterpart methods; i.e., FPCA and PCA. Such results strengthen the significance of fractional transformation in the face recognition task as a way to reduce the negative impact of undesirable variations that are present in facial images.

According to the theory of FCM, the value of the order $r$ in FCM-based methods can affect their recognition performances. Moreover, the optimal value of this parameter depends on the characteristics of a particular facial dataset. However, in this work, the values of $r$ for the ORL, Yale and Georgia Tech face datasets are set following the procedure in [25], where these values are empirically derived based on the cumulative contribution rate of the first $k$ eigenvalues.

## 5. Conclusions

In this paper, a direct extension of F2DPCA for face recognition, the IF2DPCA, is proposed. The IF2DPCA and F2DPCA methods are conceptually similar, but differ in that the former uses the fractional transformed 2D images not only for evaluating the fractional ($r$ −order) covariance matrix, but also for extracting the feature matrices. With this formulation, the redefined feature matrices capture the inherent characteristics of facial images and are relatively insensitive to undesirable variations. The experiments on the ORL, Yale and Georgia Tech face datasets demonstrate the utility of the suggested method and exhibit that in all cases, IF2DPCA outperforms F2DPCA, 2DPCA, FE, FPCA and PCA in terms of maximal and average recognition rates.

Obviously, the idea of the proposed IF2DPCA method can be easily adapted for other versions of 2DPCA. Another future work may focus on developing a sophisticated algorithm for identifying the optimal value of the fractional order.

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**ملخص البحث:**

تعدّ طريقة تحليل المكونات الرئيسية ذي البُعديْن من التّقنيات ذات الحيّز الفرعي المستخدمة في تمثيل الوجوه وتمييزها. ولربّما كان النّمط المعياري من هذه التقنية (2DPCA) غير قادرٍ على استخلاص سِمات المعلومات لِيَصِفَ على نحوٍ كافٍ المعلومات البنيوية الكامنة في الصّور الأصلية للوجوه في ظلّ وجود تغيّراتٍ في ظروف الإضاءة وتعابير الوجوه وما الى ذلك.

ولمعالجة ذلك القصور، تقترح هذه الورقة طريقة محسَّنة تقوم على التّحليل الجزئي ذي البُعديْن المستند الى تحليل المكوّنات الرئيسية (IF2DPCA)، وهي بمثابة امتداد للتّحليل الجزئي ذي البُعديْن باستخدام تحليل المكونات الرئيسية (F2DPCA) الذي تمّ تطويره بناءً على مصفوفة التّغيّرات المشتركة الجزئية (FCM). وتستخدم الطريقتان (IF2DPCA) و (F2DPCA) المبدأ نفسه لتعلُّم مصفوفة إسقاط، إلاّ أنّ الطريقة المقترحة توسّع من استخدام الأجزاء المتحولة من الصّور ذات البُعديْن عبر مهمة التّمييز بكاملها. ونتيجةً لذلك، فإنّ الحيّز الفرعي المُنَمْذَج في الطّريقة المحسّنة المقترحة يحافظ على المحتوى الذي ينطوي على أهمّ المعلومات في صُور الوجوه ذات البُعديْن، وتصبح عملية التّمييز أقلّ حساسية للتّغيُّرات في ظروف الإضاءة وتعابير الوجه وغيرها.

وقد تمّت تجربة الطّريقة المحسّنة المقترحة على ثلاثٍ من مجموعات البيانات؛ إذ أثبتت فعاليتها في تمييز الوجوه.