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AN EIGEN-APPROACH FOR JOINT ESTIMATION OF DIRECTION-OF-ARRIVAL AND FREQUENCY OF AN UNKNOWN NUMBER OF SIGNALS

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ABSTRACT

An accurate method for estimating the direction-of-arrival (DOA) jointly with the frequencies of an unknown number of source signals is proposed using the Eigen-approach. Using the minimum eigenvalues of the autocorrelation matrices produces both the DOA and the corresponding frequencies.

By moving the roots produced from the eigenvector one-by-one, the angular location is first found. The frequency is then estimated using the same procedure. Finally, the frequency is used with the angular location to estimate the DOA angle.

The results show an accurate estimation of source signals' DOA and frequency in the presence of different levels of noise.

KEYWORDS

Direction-of-arrival (DOA), Antenna arrays, Eigenvalues, Eigenvectors, Frequency estimation.

1. INTRODUCTION

Generally, the antenna array factor is designed to receive the desired signal from a particular direction while suppressing the undesired signals. Therefore, the direction of arrival (DOA) of the received source signals needs to be estimated [1].

Many DOA estimation methods have been proposed. The Eigen-approach has received wide attention, since it gives high accuracy results [2]-[6]. Most of the proposed methods needed the exact number of sources to separate signals from noise. However, the exact number of sources is typically an unknown value. Additionally, the proposed methods focused on DOA without regard to frequency estimation [2]-[4], [6].

Others have proposed different techniques to jointly estimate the DOA and the related source frequency [7]-[13]. The extended Kalman filter and unscented Kalman filter were utilized in [7] to jointly estimate the DOA and frequency of source signals. Unfortunately, high computational iterations were needed to realize good results.

Some used ESPRIT, MUSIC and Maximum Likelihood Estimation (MLE) to estimate the DOA [4], [14]-[16]. The results achieved were good, but they suffer from the complexity of the problem and the need for an exact number of sources. Although others [17] proposed methods to estimate the number of signal sources, additional computations to find the number of sources make the Eigen-approach proposed here faster, where the number of sources is found while performing other computations.

A general comparison between different DOA estimation algorithms was discussed in [18]-[19]. The methods compared in [18, 19] only discussed DOA estimation with no mention of frequency estimation, since they assume a single frequency or a known set of frequencies.

Since the DOA estimation is a nonlinear optimization problem, random search algorithms were proposed to estimate the DOA. In particular, genetic algorithm (GA) was used directly or in conjunction with other techniques to estimate the DOA [20].

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In this paper, the Eigen-approach is used to find the array factor with the coefficients of the eigenvector corresponding to the minimum eigenvalue of the autocorrelation matrix, which produces the minimum output power of the array. These coefficients are represented by the roots of the polynomial lying on the unit circle [21]. By moving the roots one by one, the proposed method estimates the DOA and the frequency using the array output.

Several simulations were carried out to show that this proposed method estimates the DOA, frequency and number of source signals accurately.

2. PROBLEM FORMULATION

Assuming an unknown number of sources (M), with different arrival angles θ_m and different frequencies f_m, transmitted to a uniform linear antenna array of $(N+1)$ elements such that $(N \geq M)$, the received signal at each array element $x_n(k)$ consists of the combination of the narrowband source signals $\vec{\boldsymbol{s}}(k)$ with additive white Gaussian noise $\vec{\boldsymbol{n}}(k)$:

$$\vec{\boldsymbol{x}}(k) = \vec{\boldsymbol{A}}.\vec{\boldsymbol{s}}(k) + \vec{\boldsymbol{n}}(k) = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\varphi_1} & e^{-j\varphi_2} & & e^{-j\varphi_M} \\ \vdots & & \ddots & \vdots \\ e^{-jN\varphi_1} & e^{-jN\varphi_2} & \cdots & e^{-jN\varphi_M} \end{bmatrix} \begin{bmatrix} s_1(k) \\ s_2(k) \\ \vdots \\ s_M(k) \end{bmatrix} + \vec{\boldsymbol{n}}(k) \tag{1}$$

where $\varphi_m = 2\pi \frac{d}{c} f_m \sin\theta_m$, d is the distance between any two consecutive antenna array elements and c is the speed of electromagnetic waves.

The signal $s_m(k)$ represents the *k*th sample of the source signal *m* and $n_n(k)$ represents the kth sample of the noise at array element *n*. The array covariance matrix can be expressed as:

$$\vec{\boldsymbol{R}}_{\boldsymbol{xx}} = \vec{\boldsymbol{A}}(\varphi)\vec{\boldsymbol{R}}_{\boldsymbol{ss}}\vec{\boldsymbol{A}}^H(\varphi) + \sigma^2\vec{\boldsymbol{I}}_{(N+1)} \tag{2}$$

where $\vec{\boldsymbol{R}}_{\boldsymbol{ss}} = E[\vec{\boldsymbol{s}}\vec{\boldsymbol{s}}^H]$ is the correlation matrix of the source signals, σ^2 is the power of the uncorrelated white Gaussian noise and $\vec{\boldsymbol{I}}_{(N+1)}$ is the identity matrix of size $(N+1)$x$(N+1)$. The output signal of the array is:

$$\vec{\boldsymbol{y}}(k) = \vec{\boldsymbol{w}}\vec{\boldsymbol{x}}(k) = \vec{\boldsymbol{w}}\vec{\boldsymbol{A}}\vec{\boldsymbol{s}}(k) + \vec{\boldsymbol{w}}\vec{\boldsymbol{n}}(k) \tag{3}$$

where $\vec{\boldsymbol{w}} = [w_1 \quad w_2 \quad \cdots \quad w_m \quad \cdots \quad w_{N+1}]$ is the weight vector of the array elements. The average output power of the array is estimated as the time average correlation of *K* samples by:

$$P_y = \vec{\boldsymbol{w}}\vec{\boldsymbol{R}}_{\boldsymbol{xx}}\vec{\boldsymbol{w}}^H \tag{4}$$

and

$$\vec{\boldsymbol{R}}_{\boldsymbol{xx}} = \frac{1}{K}\sum_{k=1}^{K} \vec{\boldsymbol{x}}(k)\vec{\boldsymbol{x}}(k)^H \tag{5}$$

When the nulls of the array factor (roots of the polynomial $((\vec{w}\vec{A}))$ on the unit circle are matched to φ_m of the source signals, the output of the array will correspond to the uncorrelated noise power only.

$$P_0 = \vec{\boldsymbol{w}}_{\boldsymbol{0}}\vec{\boldsymbol{R}}_{xx}\vec{\boldsymbol{w}}_{\boldsymbol{0}}^H = \sigma^2\vec{\boldsymbol{w}}_{\boldsymbol{0}}\vec{\boldsymbol{w}}_{\boldsymbol{0}}^H \tag{6}$$

where $\vec{\boldsymbol{w}}_{\boldsymbol{0}}$ is the weight vector which eliminates the signals at the array output.

The optimization problem is defined as follows:

$$\min_{w} \quad \vec{\boldsymbol{w}}\vec{\boldsymbol{R}}_{\boldsymbol{xx}}\vec{\boldsymbol{w}}^H \tag{7}$$

Subject to:

$$\vec{\boldsymbol{w}}\,\vec{\boldsymbol{w}}^H = 1;$$

The solution $\vec{\boldsymbol{w}}$ for the optimization problem can be found using a Lagrange multiplier; $\vec{\boldsymbol{w}}$ is a set of eigenvectors of the autocorrelation matrix R_{xx} with corresponding eigenvalues $\vec{\boldsymbol{\lambda}}$.

To minimize the objective function in Equation (7), the eigenvector $\vec{\boldsymbol{w}}_{\mathbf{0}}$ should be chosen such that it corresponds to the minimum eigenvalue λ_{min} . The corresponding minimum output power P_0 will be:

$$P_0 = \lambda_{min} = \vec{\boldsymbol{w}}_{\mathbf{0}}^H \vec{\boldsymbol{R}}_{\boldsymbol{xx}} \vec{\boldsymbol{w}}_{\mathbf{0}} = \sigma^2 \tag{8}$$

It was shown in [21] that the roots of the array polynomial made by $\vec{\boldsymbol{w}}_{\mathbf{0}}$ coincided with φ_m for minimum output power. If w_{0n} represents the n^{th} element of the eigenvector $\vec{\boldsymbol{w}}_{\mathbf{0}}$, the array factor can then be written as:

$$F(f,\theta) = \sum_{n=0}^{N} w_{0n} e^{-jn\varphi} = \sum_{n=0}^{N} w_{0n} e^{-j2n\pi\frac{d}{c}f\sin\theta} = \prod_{n=1}^{N} w_{0N} \left(e^{-j2\pi\frac{d}{c}f\sin\theta} - e^{-j\hat{\varphi}_n} \right) \tag{9}$$

The above equation shows that $\hat{\varphi}_n$ match φ_m of the signals as the solution is achieved for the minimum output power.

Similarly, an adaptive FIR filter of order (L) and weight vector $\vec{\boldsymbol{\eta}}$ is used to determine the frequency content of the output signal of the array. The transfer function of the FIR filter is:

$$\frac{Z(f)}{Y(f)} = H(f) = \sum_{l=0}^{L} \eta_l \, e^{-jl\psi} = \prod_{l=1}^{L} \eta_L \left(e^{-j2\pi\frac{f}{f_s}} - e^{-j\hat{\psi}} \right) \tag{10}$$

where $\psi_m = 2\pi \frac{f_m}{f_s}$ and f_s is the sampling frequency.

The output of the filter is:

$$\vec{\boldsymbol{z}}(k) = \sum_{l=0}^{L} \eta_l \, \vec{\boldsymbol{y}}(k-l) = \vec{\boldsymbol{\eta}} \begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-L) \end{bmatrix} = \vec{\boldsymbol{\eta}} \vec{\boldsymbol{y}}(l,k). \tag{11}$$

The average output power of the filter can be estimated as:

$$P_z = \vec{\boldsymbol{z}} \vec{\boldsymbol{z}}^H = \vec{\boldsymbol{\eta}} \vec{\boldsymbol{R}}_{\boldsymbol{yy}} \vec{\boldsymbol{\eta}}^H \tag{12}$$

where,

$$\vec{\boldsymbol{R}}_{\boldsymbol{yy}} = \frac{1}{K} \sum_{k=1}^{K} \vec{\boldsymbol{y}}(l,k) \vec{\boldsymbol{y}}(l,k)^H . \tag{13}$$

When the nulls of $H(f)$ on the unit circle are matched to ψ_m of $Y(f)$, the output of the filter will correspond to the uncorrelated noise power only.

$$P_{z_0} = \vec{\boldsymbol{\eta}}_{\mathbf{0}} \vec{\boldsymbol{R}}_{\boldsymbol{yy}} \vec{\boldsymbol{\eta}}_{\mathbf{0}}^H = \sigma_1^2 \vec{\boldsymbol{\eta}}_{\mathbf{0}} \vec{\boldsymbol{\eta}}_{\mathbf{0}}^H \tag{14}$$

where $\vec{\boldsymbol{\eta}}_{\mathbf{0}}$ is the weight vector which eliminates the signals at the filter output and σ_1^2 is the power of the noise signal at the input of the filter.

Similarly, the optimization problem is defined as:

$$\min_{\eta} \quad \vec{\boldsymbol{\eta}} \vec{\boldsymbol{R}}_{\boldsymbol{yy}} \vec{\boldsymbol{\eta}}^H \tag{15}$$

subject to

$$\vec{\boldsymbol{\eta}} \vec{\boldsymbol{\eta}}^H = 1$$

The eigenvector η_0 that corresponds to the minimum eigenvalue yields the minimum output power as in Equation (14) and is related to ψ_m as in Equation (10).

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3. DOA AND FREQUENCY ESTIMATION

The process of frequency and DOA estimation can be obtained by calculating the pseudo-spectrum at the angles and frequencies corresponding to the polynomial roots as in Equations (9, 10). The pseudo-spectrum is calculated by altering each root of the array factor and the FIR filter to obtain the corresponding output power of the array and the filter. Large variation of the output power will occur if the roots coincide with an actual angle and frequency of a signal; otherwise, this root does not correspond to any of the source signals.

The method for estimating the frequency and DOA is as follows:

(1) The eigenvalues and the eigenvectors of $\overrightarrow{\boldsymbol{R}_{xx}}$ are calculated.

(2) An eigenvector ($\overrightarrow{\boldsymbol{w}_0}$), corresponding to the minimum eigenvalue (λ_{min}), is used in Equation (9) to calculate $\widehat{\boldsymbol{\varphi}}_{\mathbf{0}} = [\hat{\varphi}_1, \hat{\varphi}_2, \dots, \hat{\varphi}_N]$ on the unit circle.

(3) The power of the uncorrelated noise is calculated as in Equation (8).

(4) For $n = 1,2, \dots, N$:

(a) Shifting one $\hat{\varphi}_n$ on the unit circle by π; i.e., $\left(\hat{\varphi}_{n,new} = \hat{\varphi}_{n,old} + \pi\right)$ and computing the corresponding weight vector $\overrightarrow{\boldsymbol{w}_n}$ using Equation (9).

(b) The output power $P_y(n)$ is calculated as in Equation (4).

(c) If $P_y(n) \le P_0$, go to step (4-a).

(d) The output signal is calculated as in Equation (3).

(e) The eigenvalues and the eigenvectors of $\overrightarrow{\boldsymbol{R}_{yy}}$ (Equation (13)) are calculated.

(f) The eigenvector ($\overrightarrow{\boldsymbol{\eta}_0}$), corresponding to the minimum eigenvalue, is used in Equation (10) to calculate $\widehat{\boldsymbol{\psi}}_{\mathbf{0}} = [\hat{\psi}_1, \hat{\psi}_2, \dots, \hat{\psi}_L]$ on the unit circle.

(g) For $l = 1,2, \dots, L$:

a. Shifting one $\hat{\psi}_l$ on the unit circle by π; i.e., $\left(\hat{\psi}_{l,new} = \hat{\psi}_{l,old} + \pi\right)$ and computing the corresponding weight vector $\overline{\eta}_l$ using Equation (10).

b. The output power $P_z(l)$ is calculated as in Equation (12).

c. If $P_z(l) \le P_{z_0}$, go to step (4-g).

d. Set values for pseudo-spectrum plot as:

$$\hat{P}_z(l) = P_z(l) - P_{z_0}$$

$$\hat{f}(l) = \frac{\hat{\psi}_{l,old}}{2\pi/f_s}$$

$$\hat{\theta}(l) = \sin^{-1}\left[\frac{\hat{\varphi}_{n,old}}{2\pi \frac{d}{c} \hat{f}(l)}\right]$$

e. Go to step (4-g)

f. Go to step (4)

g. Plot pseudo spectrum $\widehat{P_z}(l) = F(\hat{f}(l)/f_s, \hat{\theta}(l))$

4. SIMULATION AND RESULTS

An array of 11 elements and an inter-element spacing of $d = {}^{c}/_{f_s}$ was chosen to simulate the proposed method. Since the number of roots is one less than the number of elements, the number of roots will be ten. This means that this antenna array can be used to estimate the location and frequency of up to ten signal sources.

Results are presented for two simulation examples by implementing the algorithm proposed in the previous section. In the first example, the noise power was -10dBm with seven narrowband source signals transmitting to the array with normalized frequencies $f = (f_m/f_s) = \{0.26946, 0.2, 0.1, 0.3, 0.35, 0.15, 0.4\}$ and angles $\theta = \{40, 60, 50, 50, -15, -30, \ -22\}$.

Note that:

$$\psi_1 = \psi_2$$

$$f_1 \sin\theta_1 = f_2 \sin\theta_2$$

and that,

$$\theta_3 = \theta_4$$

to show the capabilities of the proposed method to resolve signals that appear to have the same location.

The results are shown in Figure 1 and Table 1. The actual signal power can be evaluated by subtracting the estimated noise level from the level of power at each frequency in the pseudo-spectrum. Table 1 shows the power levels, the estimated DOA and the estimated frequency for the seven source signals. The results show that the proposed method was able to estimate the angle, the frequency and power level of each source signal accurately without knowing the number of source signals.

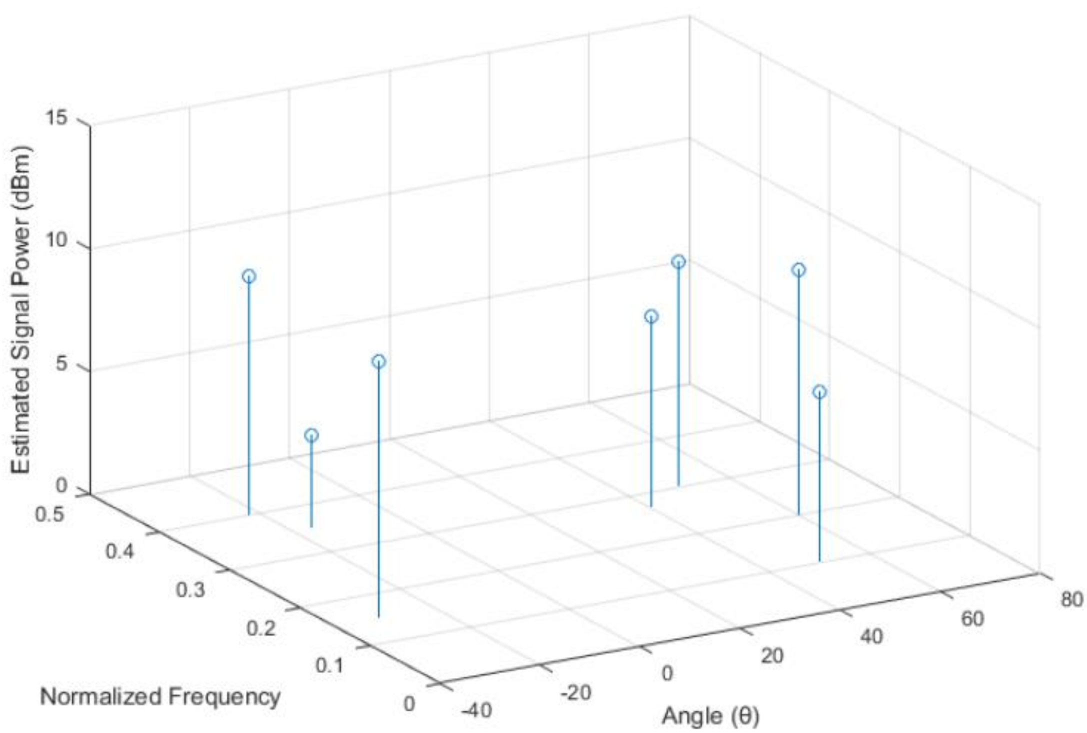

Figure 1. The estimated frequency and angle in the pseudo-spectrum for seven source signals. The source signals are transmitted with normalized frequencies of 0.26946, 0.2, 0.1, 0.3, 0.35, 0.15 and 0.4 at angles of 40º, 60º, 50º, 50º, -15º, -30º and -22º, respectively with different power levels.

Table 1. The estimated angles $\hat{\theta}_i$, power levels $\hat{P}_i$ and frequency $\hat{f}_i$ using the proposed method. The number of source signals is 6, the number of array elements is 11 and the noise power is -10dBm.

θ_i	$P_i\ (dBm)$	f_i	$\hat{\theta}_i$	$\hat{P}_i\ (dBm)$	$\hat{f}_i$
40	7.7815	0.26946	39.9599	7.7767	0.26946
60	10	0.2	59.9163	9.9999	0.2
50	6.9897	0.1	49.8815	6.9893	0.1
50	9.0309	0.3	49.8968	9.1995	0.3
-15	6.9897	0.35	-16.536	3.8268	0.3504
-30	9.0309	0.15	-31.318	10.4606	0.1502
-22	10	0.4	-22.203	9.7626	0.3998

For the second example, the noise power was increased to 7dBm to show the effect of the noise on the proposed method. The results are shown in Table 2.

Table 2. The estimated angles $\hat{\theta}_i$, power levels $\hat{P}_i$ and frequency $\hat{f}_i$ using the proposed method. The number of source signals is 6, the number of array elements is 11 and the noise power is 7dBm.

θ_i	$P_i\ (dBm)$	f_i	$\hat{\theta}_i$	$\hat{P}_i\ (dBm)$	$\hat{f}_i$
40	7.7815	0.26946	42.44	7.293	0.2706
60	10	0.2	66.04	9.14	0.1998
50	6.9897	0.1	53.33	7.885	0.1005
50	9.0309	0.3	50.13	8.947	0.2997
-15	6.9897	0.35	-13.05	5.731	0.3507
-30	9.0309	0.15	-31.83	9.284	0.1501
-22	10	0.4	-22.26	9.884	0.4007

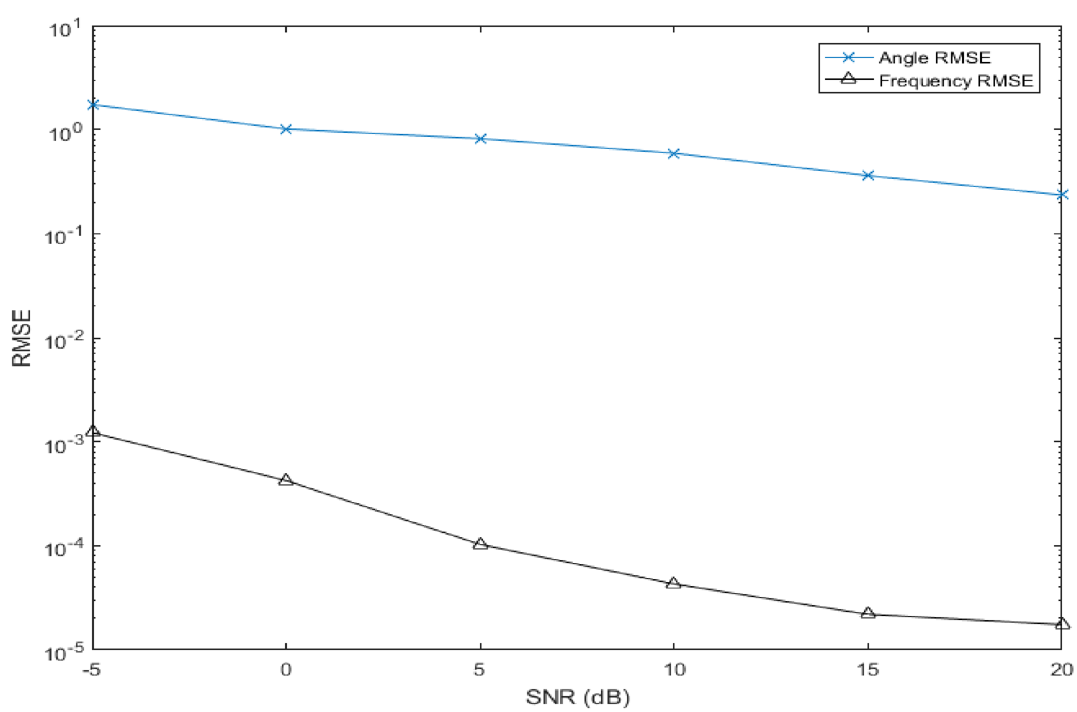

Figure 2. The root mean square error in estimating the angle $\hat{\theta}_i$ and the frequency $\hat{f}_i$ using the proposed method for different SNR levels.

5. CONCLUSION

The direction-of-arrival angles and the source signals' frequencies were estimated using Eigen-approach with no prior knowledge of the number of signals. The proposed method first found the minimum eigenvalue of the autocorrelation matrix of the array elements input signals. The eigenvector, which corresponds to the minimum eigenvalue, represents the weights of the array factor. The output of the first stage yields the values of the angular location $(f \sin\theta)$, while the output of the second stage yields the source signal frequencies which are used to find the DOA angles using the angular locations from the first stage. The proposed method was able to handle different levels of noise to effectively find the DOA angle and source signal frequency.

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ملخص البحث:

تم اقتراح طريقة دقيقة لتقدير اتجاه الوصول جنباً الى جنب مع الترددات لعدد غير معلوم من الإشارات من مصادر مختلفة. وتستخدم الطريقة المقترحة التي تعتمد النهج الذاتي أقّل القيم الذاتية لمصفوفات الارتباط الذاتي من أجل الحصول على كل من اتجاهات الوصول والترددات المناظرة لها. وبتحريك الجذور المتولّدة من المتّجه الذاتي واحداً تلو الآخر، يتم أولاً إيجاد الموقع الزاوية لتقدير زاوية اتجاه الوصول. وتبين التقنية المقترحة تقديراً دقيقاً لاتجاهات وصول الإشارات وتردداتها في وجود مستويات مختلفة من الضجيج.